Drifting beyond Bayesics

A Bayesian Implementation of the Circular Drift Diffusion Model

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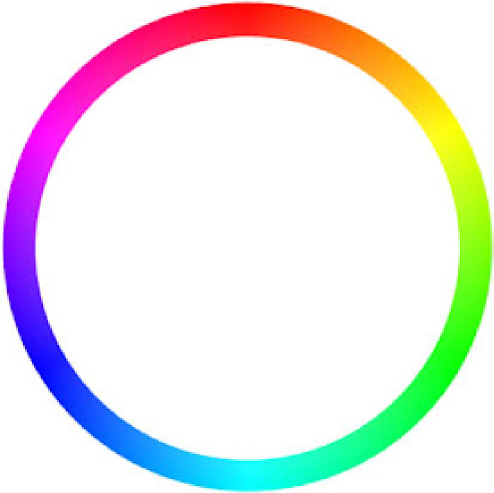
Some Circular Decisions

Indicate the Color

What is the color of the shirt?

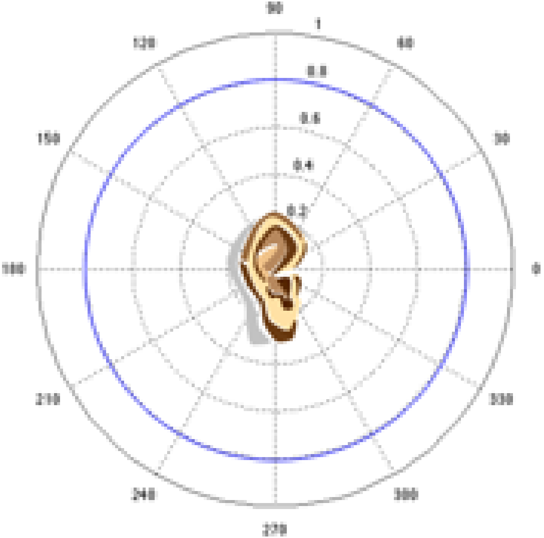
Did You Remember the Color?

What was the color of the shirt?



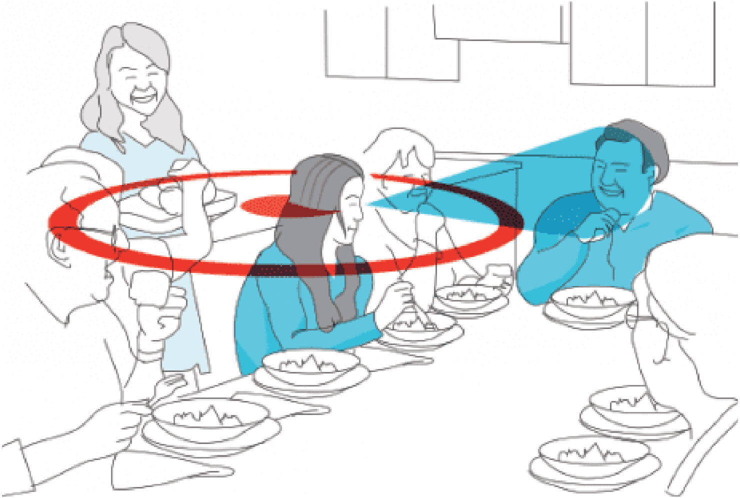
Spatial Identification of Sound

Testing a directional hearing aid



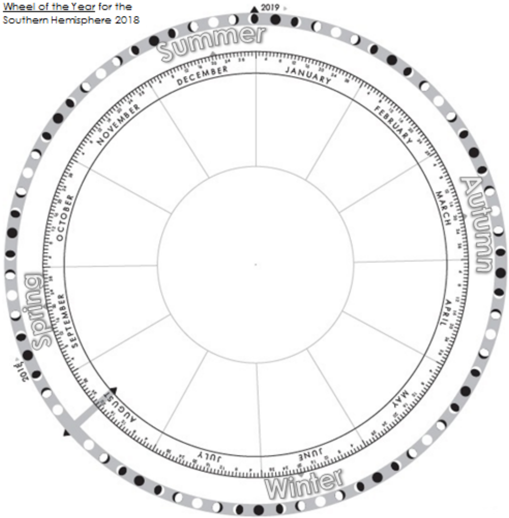
Conversation Source

Where is the conversation coming from?



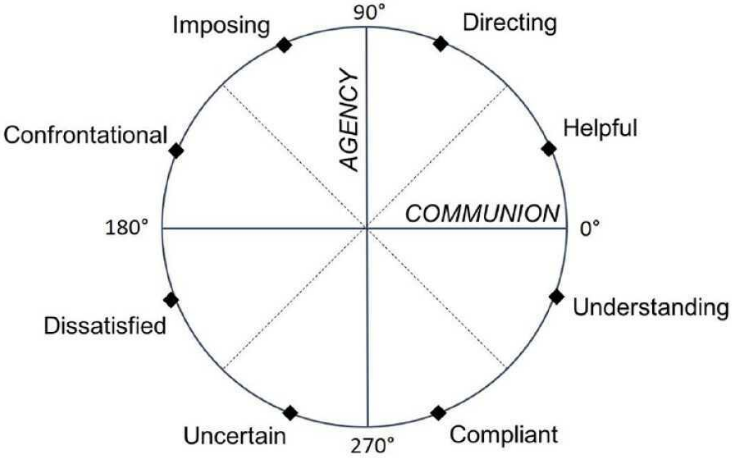
Predicting Weather

Which day will have the highest maximum temperature in Sydney?



Assessing Personalities

What is this person's personality?



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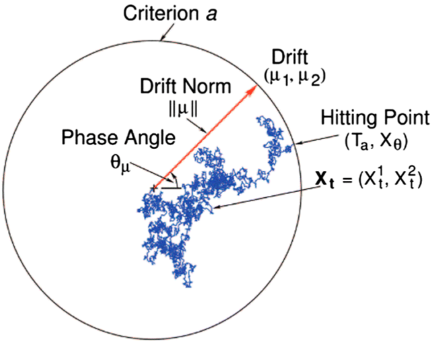
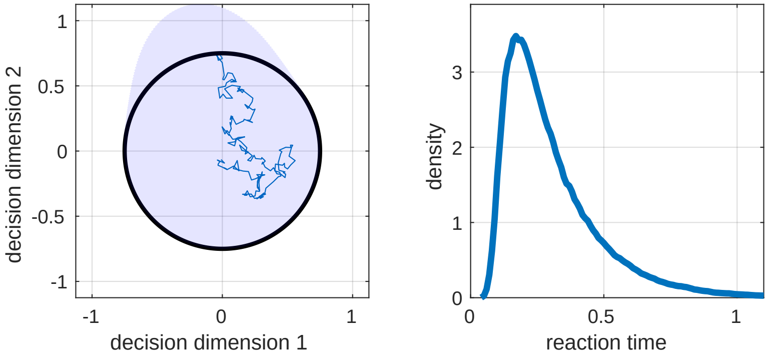


The Circular Drift Diffusion Model (CDDM)

Given the parameters, CDDM predicts a distribution of angles and reaction times



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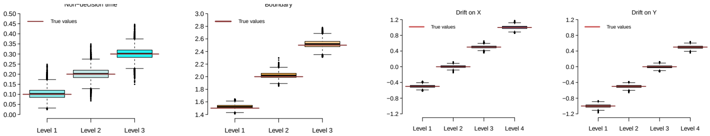
  - allows for flexible and rapid model development, including hierarchical and latent-mixture structures
  - automates fully Bayesian inference via computational methods
- We conducted simulation studies and found good parameter recovery even with small sample sizes ($N = 80$)



JAGS Model Design Patterns

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- Likelihood function

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y[time,1:2] ~ dcddm(delta[PERSON[time], DIFFICULTY[time]],
                    eta[PERSON[time], SPEED_ACCURACY[time]],
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- Prior distribution: latent mixture of angles

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latent_state[time] ~ dbern(omega[PERSON[time], CUE_DEFLECT[time]])
theta[time,1]      ~ dnorm(POSITION[time], ... )
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- Hierarchical distribution: drift

```
for(dIdx in 1:nDifficulty){
   mu_delta[dIdx] ~ dnorm(0, 1)  # Prior on conditional means
   for(pIdx in 1:nParticipants){
      log_delta[pIdx, dIdx] ~ dnorm(mu_delta[dIdx], tau_delta)
      delta[pIdx,dIdx]      = exp(log_delta[pIdx, dIdx])
   }
}
```

An Application

Kvam (2019) Task

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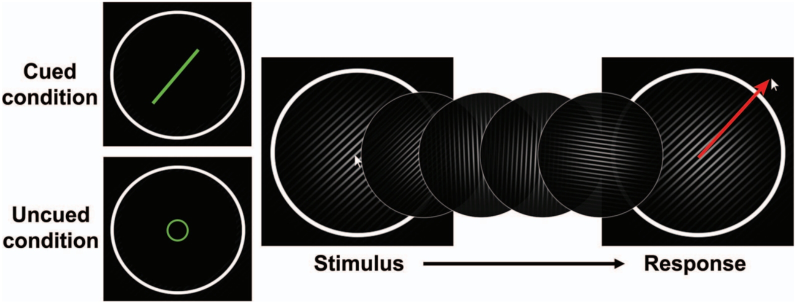

Research questions

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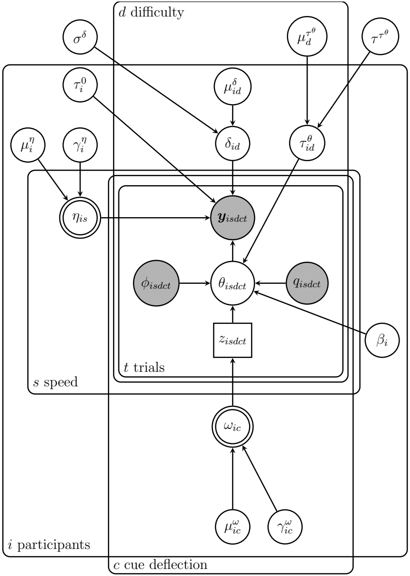
2. Is the speed of information processing less for more variable stimuli?
3. Do people get information less consistently from more variable stimuli?
4. Are there differences in being influenced by the cue for different cue angles?

Model



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\begin{aligned}
\mu_i^{\eta} &\sim \text{Gaussian}(0,1) \\
\gamma_i^{\eta} &\sim \text{Gaussian}(0,1)T(0,\infty) \\
\eta_{is} &= \begin{cases} \exp\left(\mu_i^{\eta} + \frac{\gamma_i^{\eta}}{2}\right) & \text{if } s = \text{accuracy} \\ \exp\left(\mu_i^{\eta} - \frac{\gamma_i^{\eta}}{2}\right) & \text{if } s = \text{speed} \end{cases} \\
\tau_i^0 &\sim \text{uniform}(0, \min y_{i1}) \\
\mu_{id}^{\delta} &\sim \text{Gaussian}(0,1) \\
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\mu_d^{\tau^{\theta}} &\sim \text{Gaussian}(0,1) \\
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z_{isdct} &\sim \text{Bernoulli}\left(\omega_{ic}\right) \\
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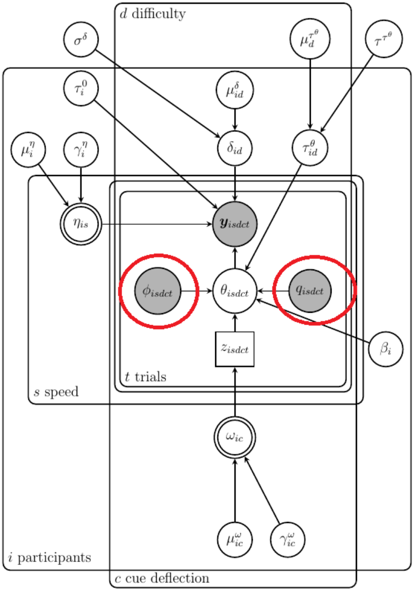

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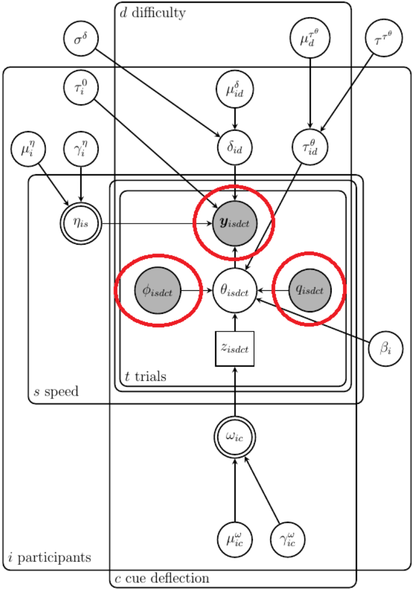

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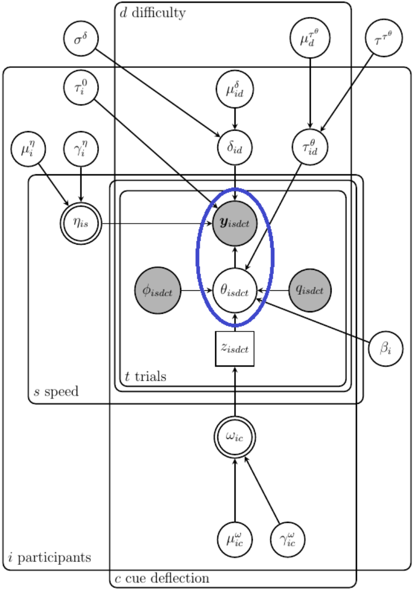

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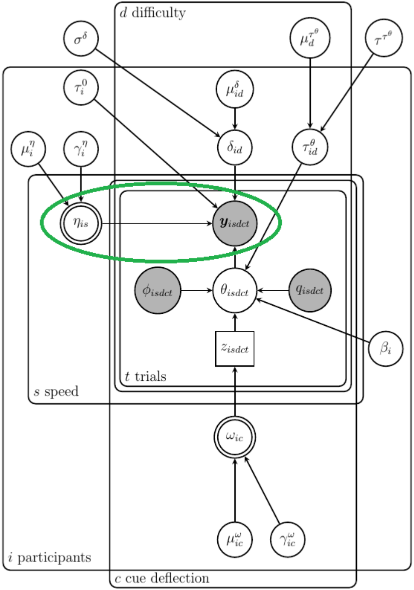

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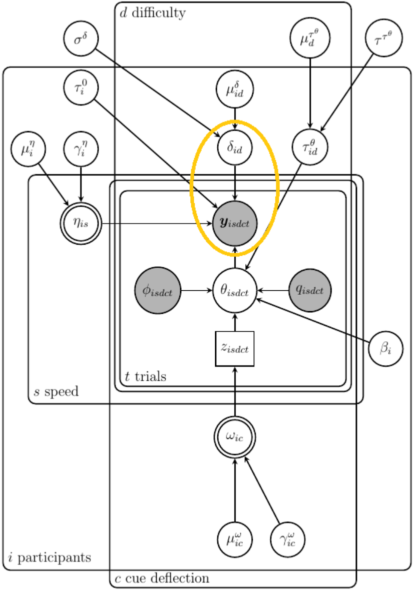

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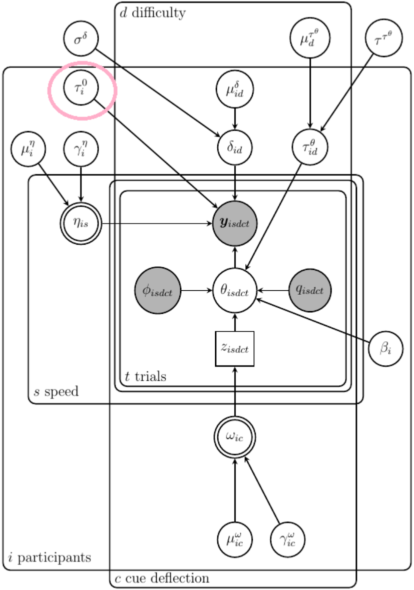

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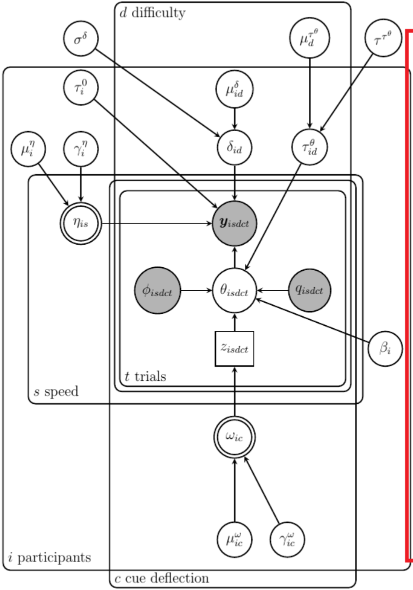

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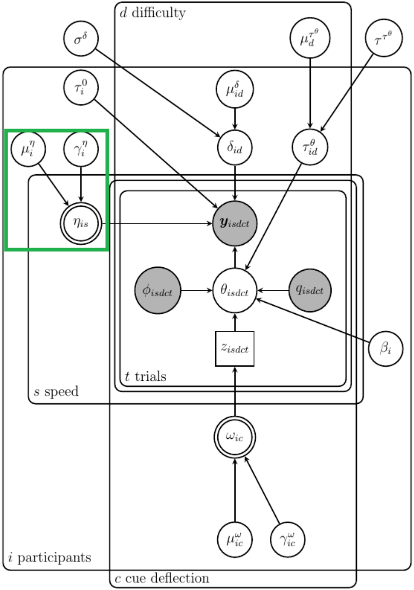

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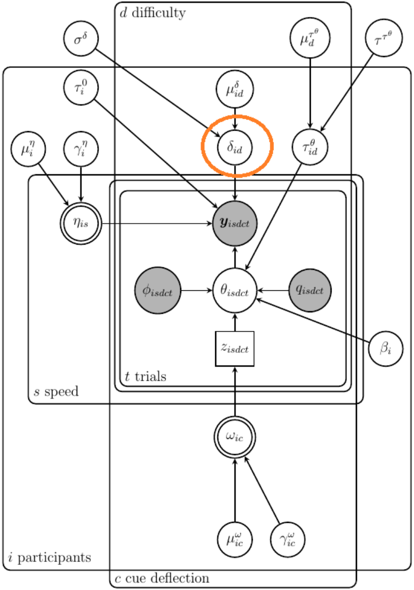

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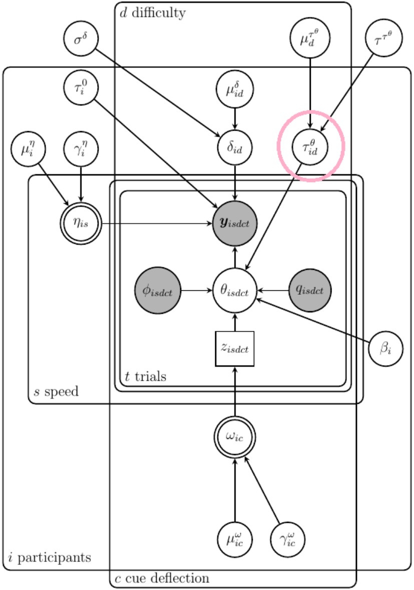

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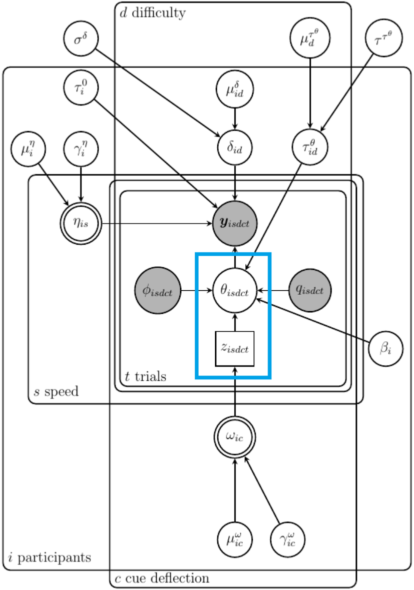

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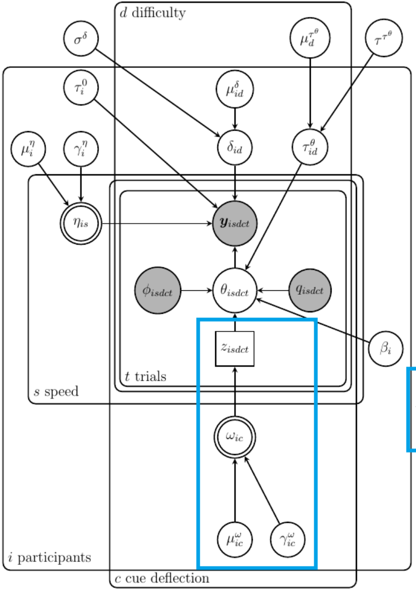

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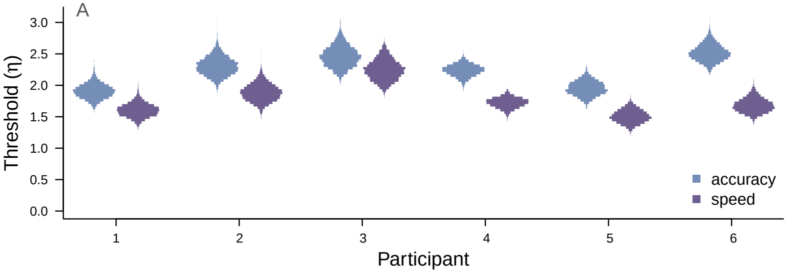

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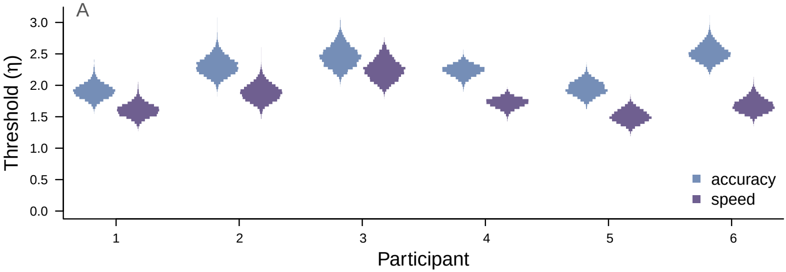

- Accuracy thresholds are significantly different (and larger), with Bayes factors above 1,000 for all but participant 3, who has a Bayes factor favoring ‘different’ of 9

Speed of Information Processing

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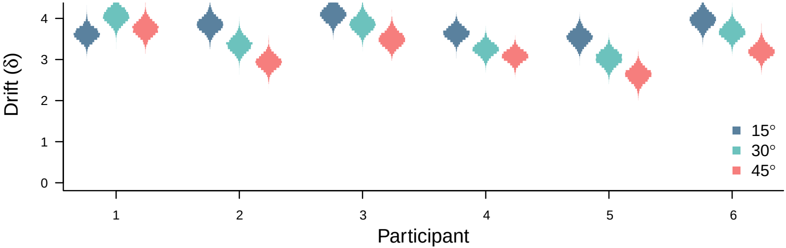

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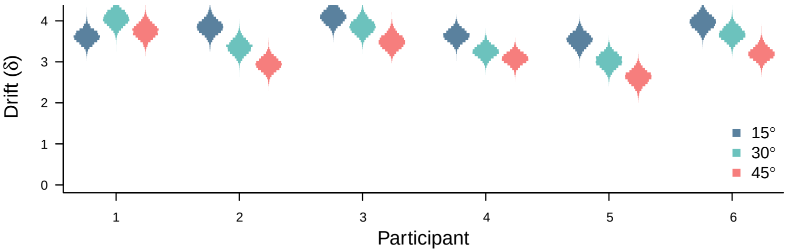

- Ordering of δ generally shows greater difficulty with more variability
  - participant 1 has lower δ than is expected for the easiest 15° stimuli

Consistency of Information Processing

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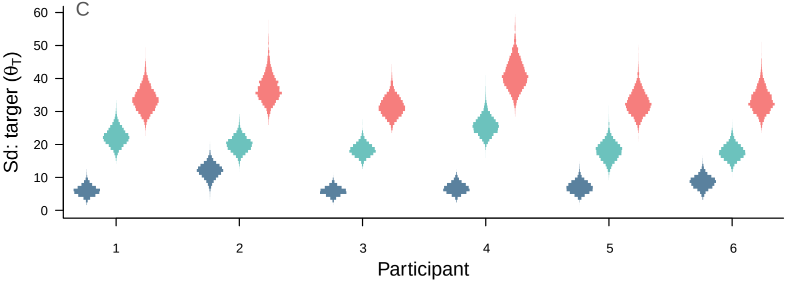

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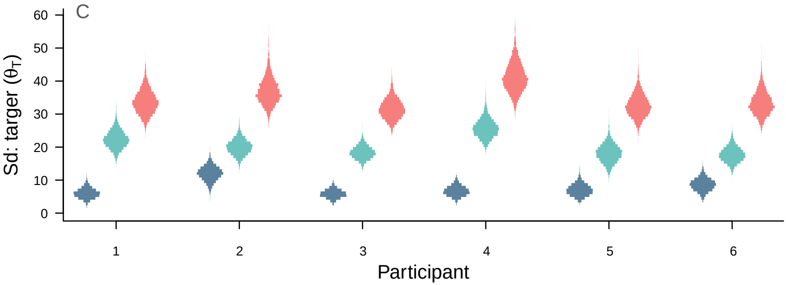

- Ordering shows less drift rate consistency as stimuli become more difficult via increased variability

Influence of Cues

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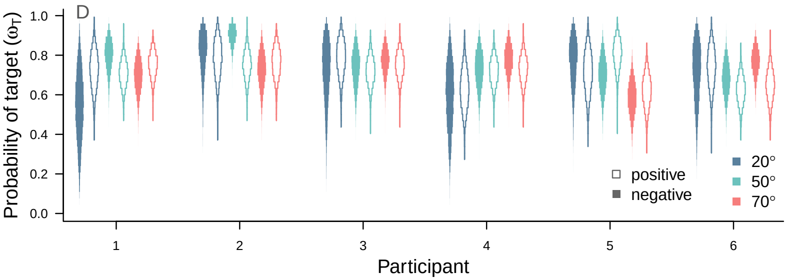


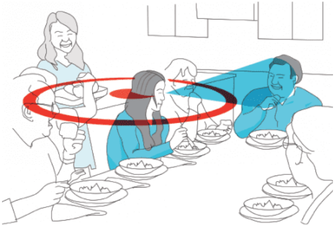
- Participants mostly ignore the cue and there are no significant differences in the base-rate for different (positive and negative) cue angle displacements

Discussion

Future work

Name the color of the shirt?

Who is talking?



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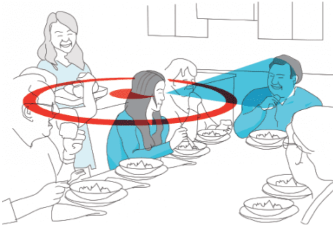

In practice, speeded orientation responses are often recorded with a discrete set of response options

Future work: A Thurstonian extension

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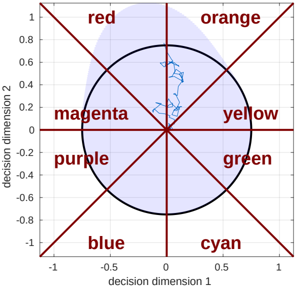

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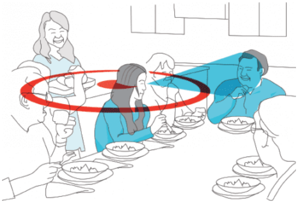

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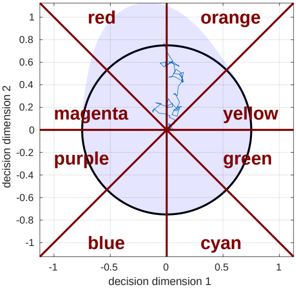

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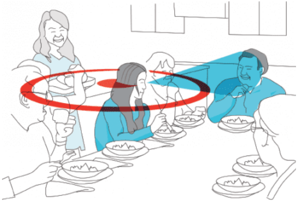

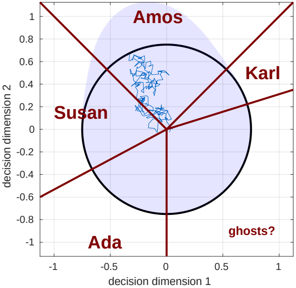

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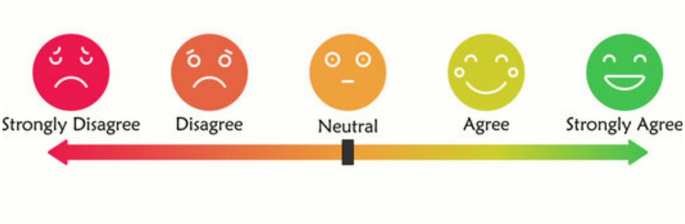

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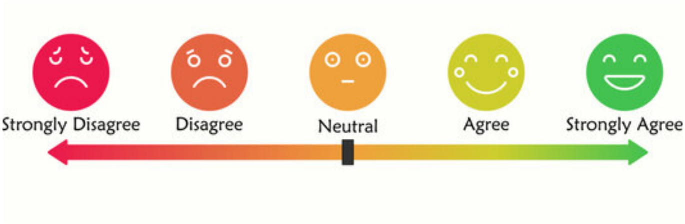

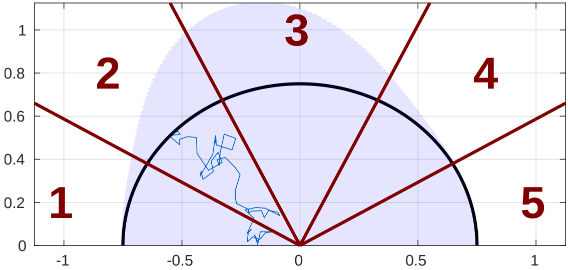

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Drifting beyond Bayesics

A Bayesian Implementation of the Circular Drift Diffusion Model

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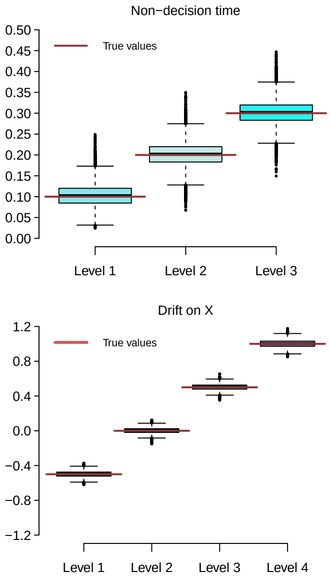
Michael D. Lee, Joachim Vandekerckhove

University of California, Irvine

Recovery Study for Cartesian Implementation



Recovery Study for Polar Implementation