A Bayesian Implementation of the Circular Drift Diffusion Model

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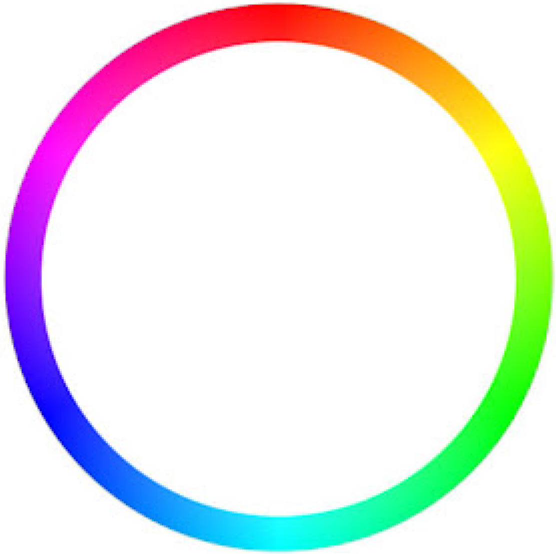
Overview

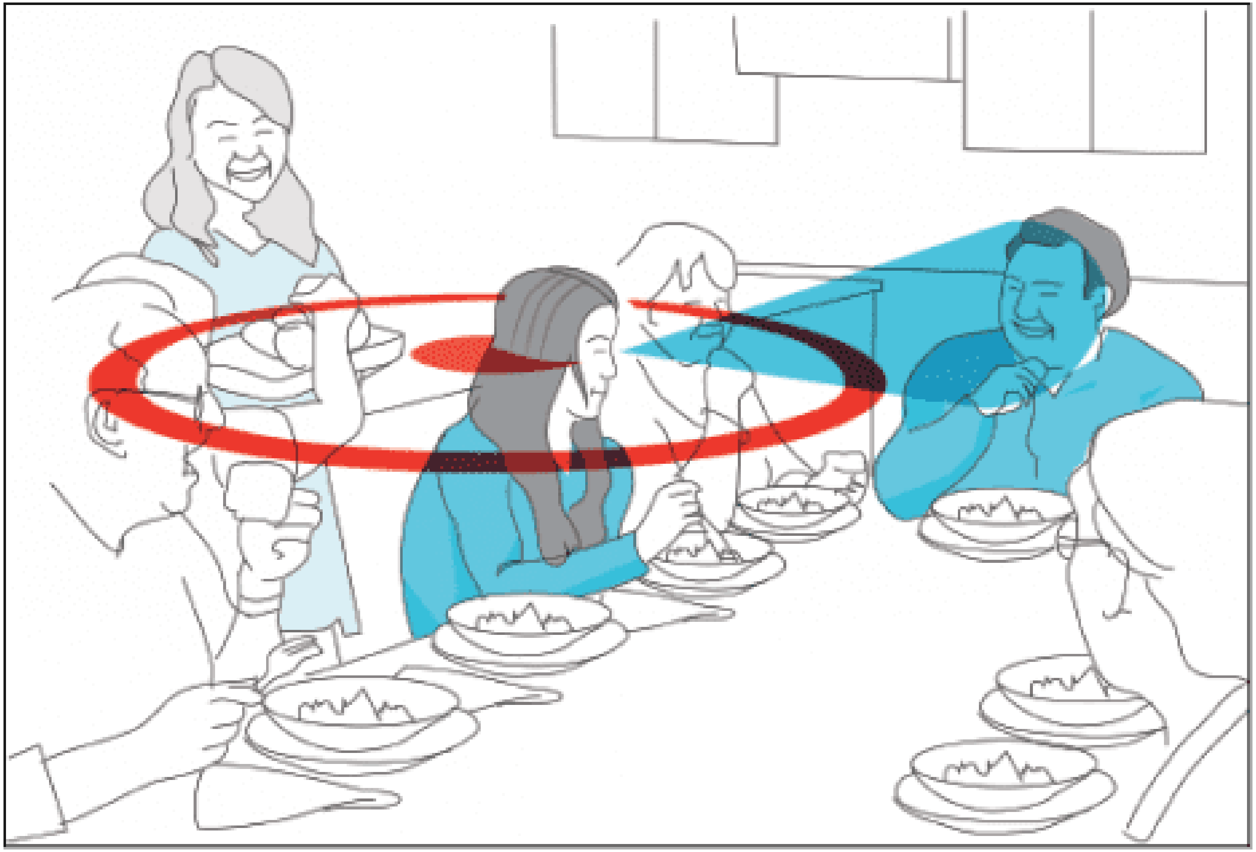
- The Circular Drift Diffusion Model (CDDM) is an extension of the drift-diffusion model to decision tasks with a circular decision space.
- We developed a custom JAGS module that allows for the implementations of the CDDM in a Bayesian framework.
- We demonstrate the adequacy of our CDDM module in simulation studies.
- We illustrate the advantages of our Bayesian implementation by revisiting publicly available data from a continuous orientation judgment task.

Decisions on a circle

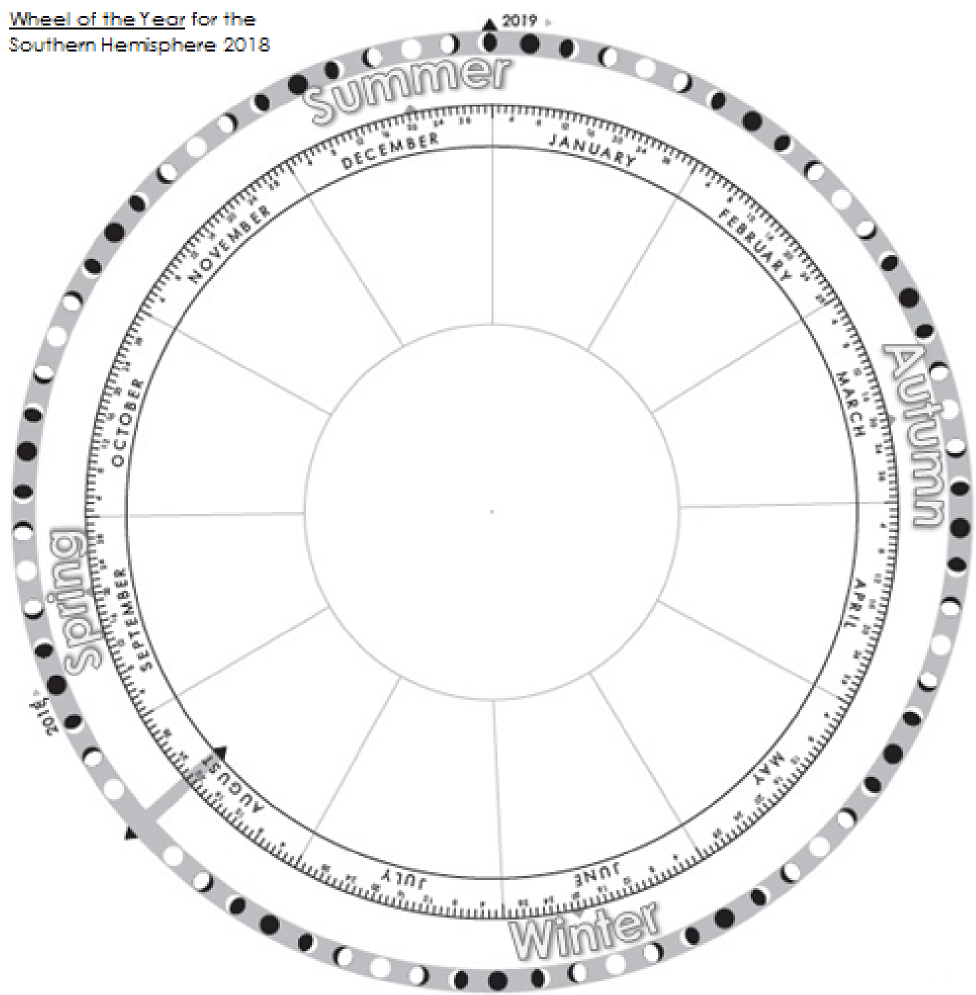
Color identification



Spatial location of sound

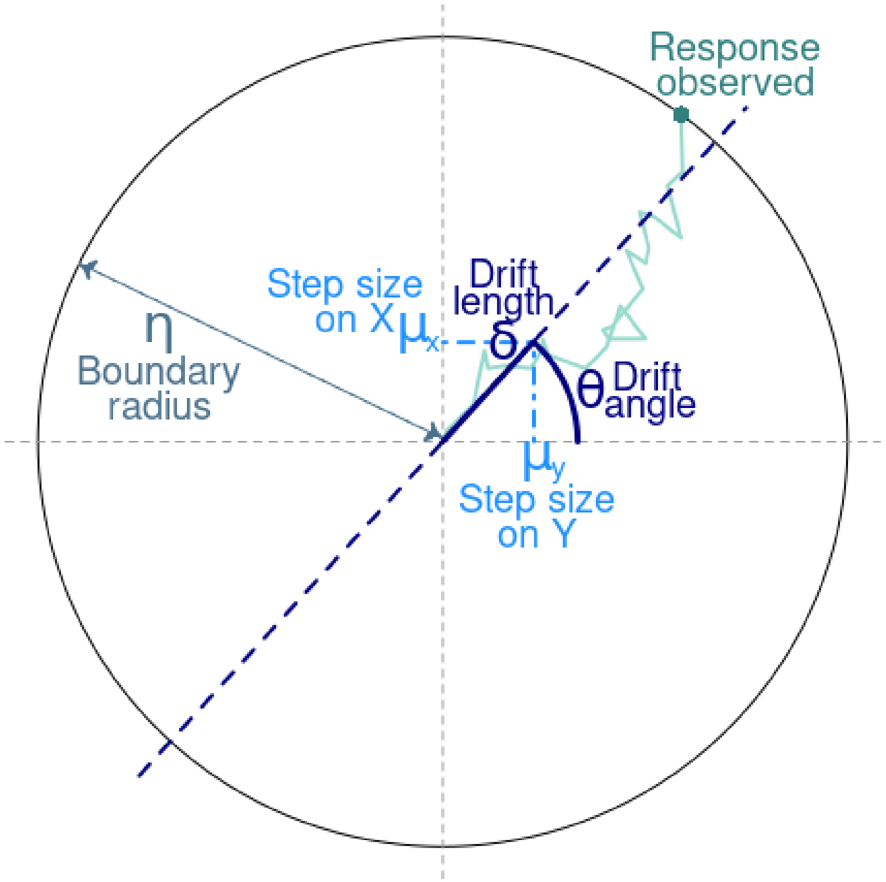


Cyclic events

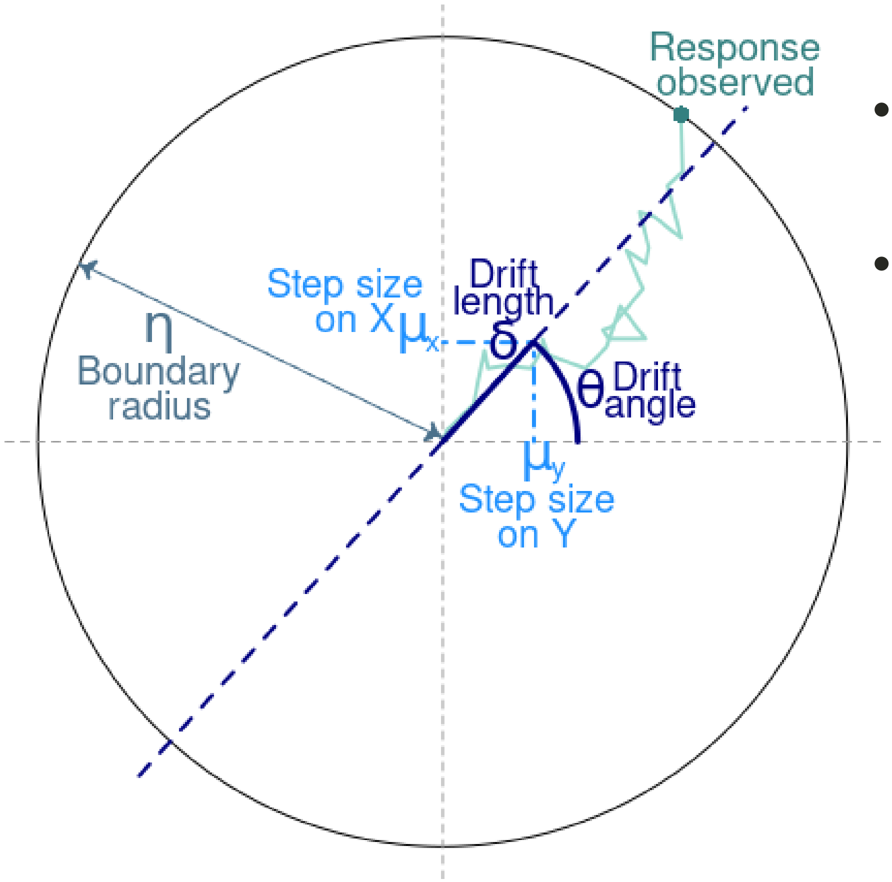


The Circular Drift Diffusion Model (CDDM)

Circular Drift Diffusion Model

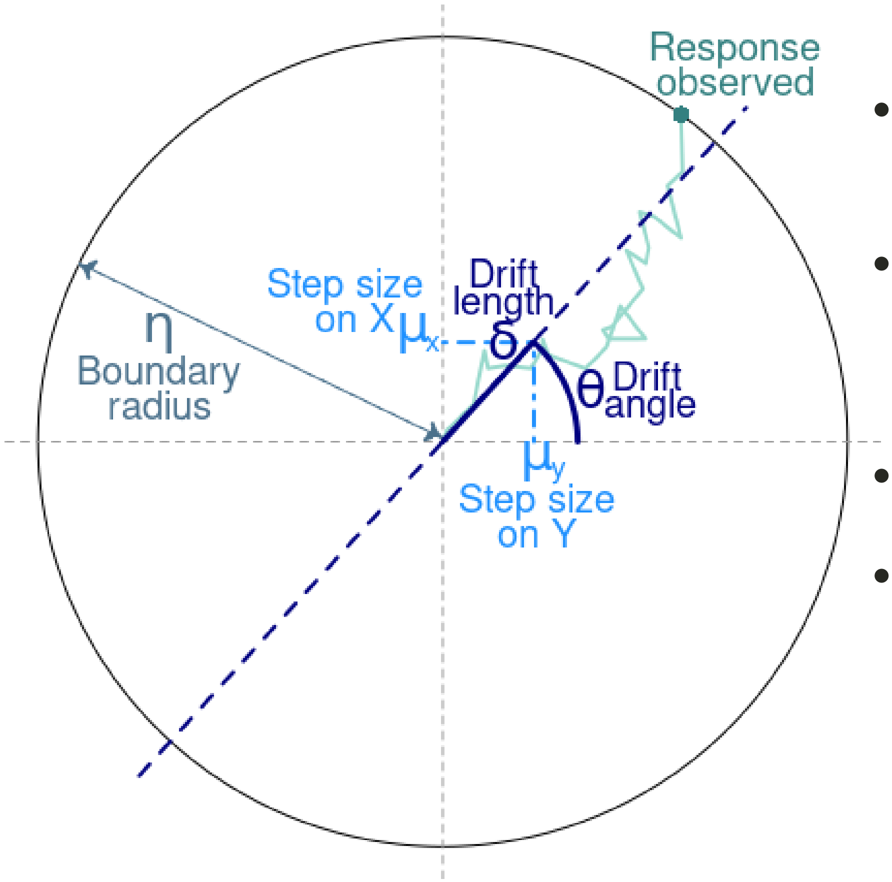


Circular Drift Diffusion Model



- Nondecision time (τ): Visual encoding and motor control.
- Boundary radius (η): Criterion to be reached to make a decision.

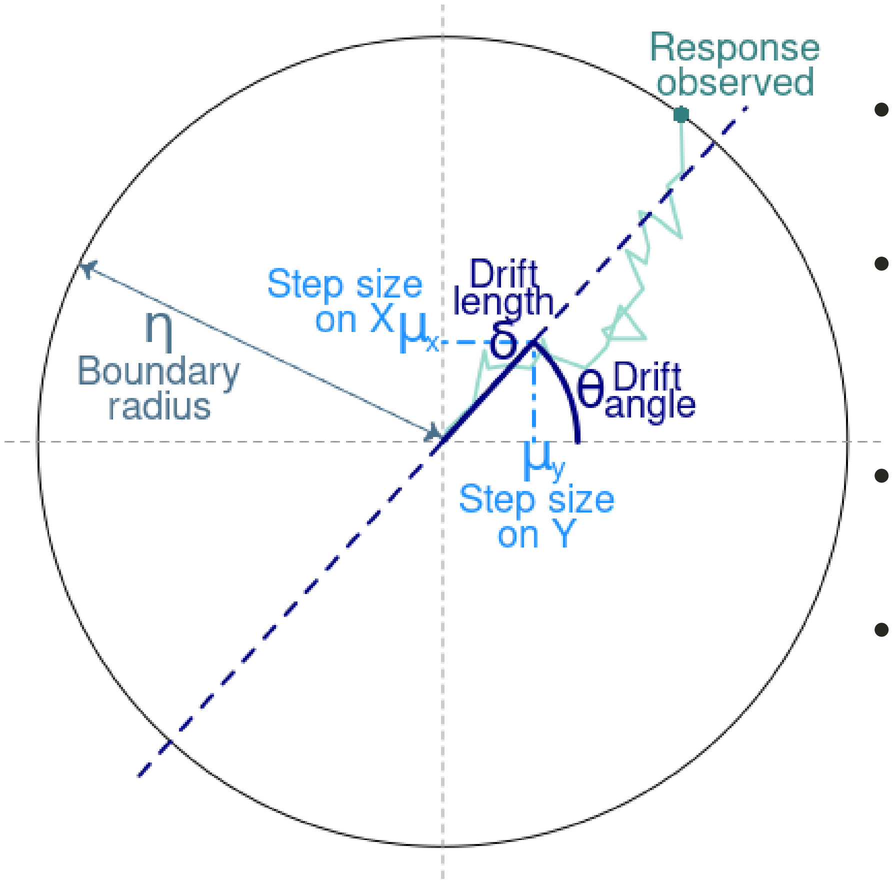
Circular Drift Diffusion Model



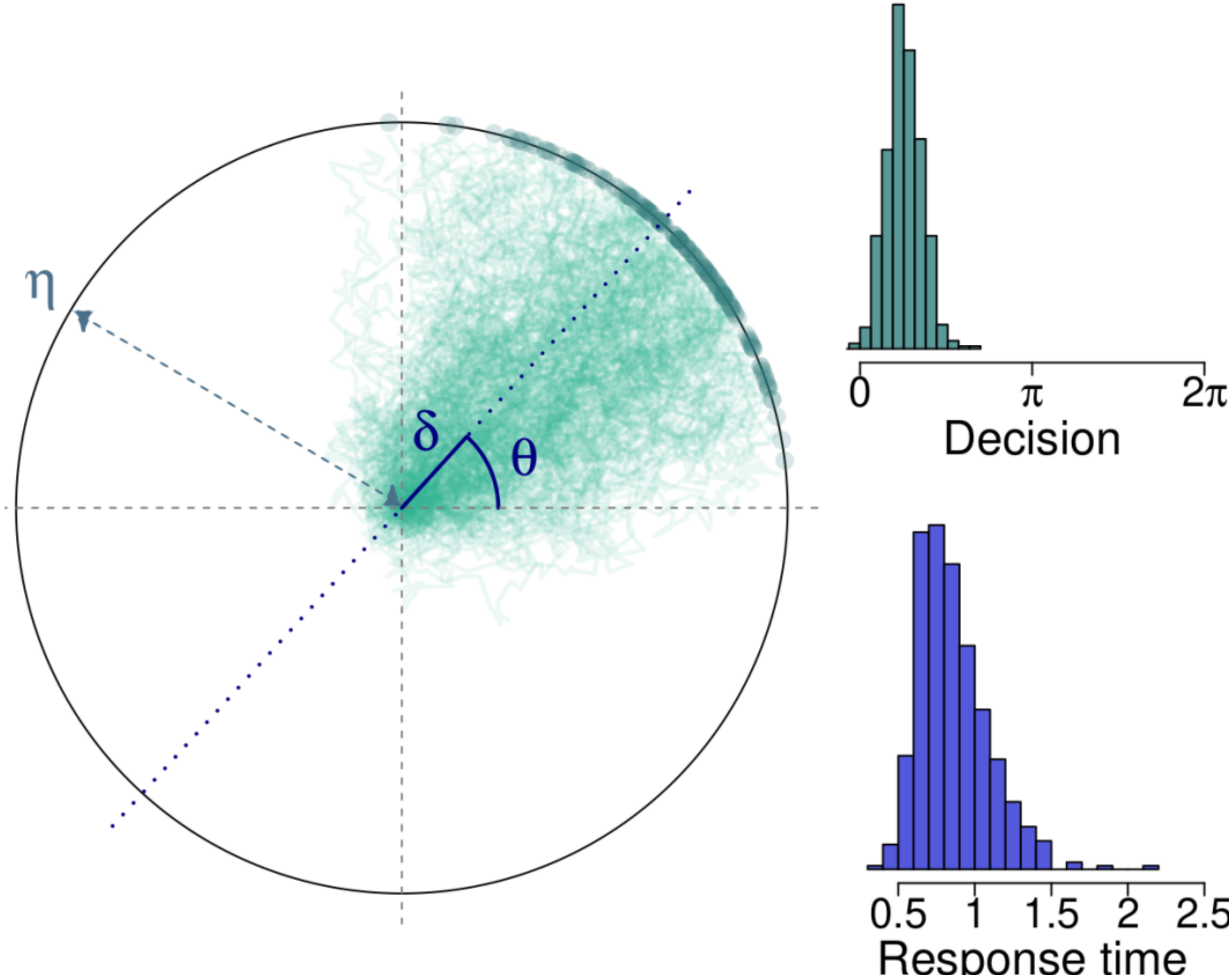
- Nondecision time (τ): Visual encoding and motor control.
- Boundary radius (η): Criterion to be reached to make a decision.
- Mean step-size on x-axis (μ_x).
- Mean step-size on Y-axis (μ_y).

$$\mu = \{\mu_x, \mu_y\}$$

Circular Drift Diffusion Model



- Nondecision time (τ): Visual encoding and motor control.
- Boundary radius (η): Criterion to be reached to make a decision.
- Drift angle (θ): Direction of stimulus evidence.
- Drift length (δ): Speed of information processing .


η
δ
θ
0
π
2π
Decision
0.5
1
1.5
2
2.5
Response time

Implementing the CDDM module in JAGS

CDDM module in JAGS

Polar coordinate implementation

```
for (i in 1:N) {
     X[1:2,i] ~ dcddmpolar(delta, theta, eta, tau)
}
```

Cartesian coordinate implementation

```
for (i in 1:N) {
     X[1:2,i] ~ dcddmcartn(mux, muy, eta, tau)
}
```

https://github.com/joachimvandekerckhove/jags-cddm

Parameter recovery

$$\eta \in \{1.5, 2.0, 2.5\}$$

$$\tau \in \{0.1, 0.2, 0.3\}$$

Cartesian implementation

$$\mu_x \in \{-0.5, 0, 0.5, 1\}$$

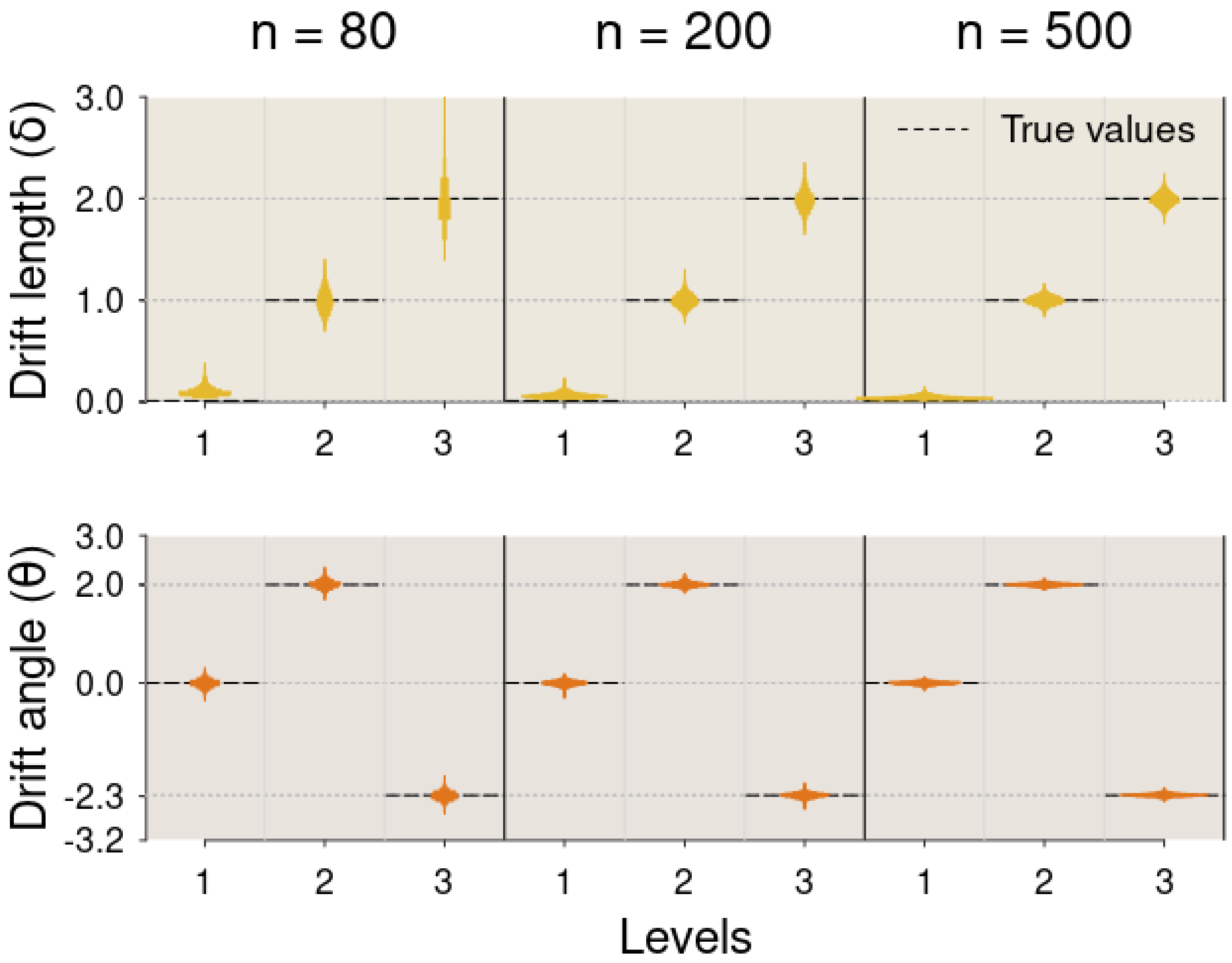
$$\mu_y \in \{-1, -0.5, 0, 0.5\}$$

Polar implementation

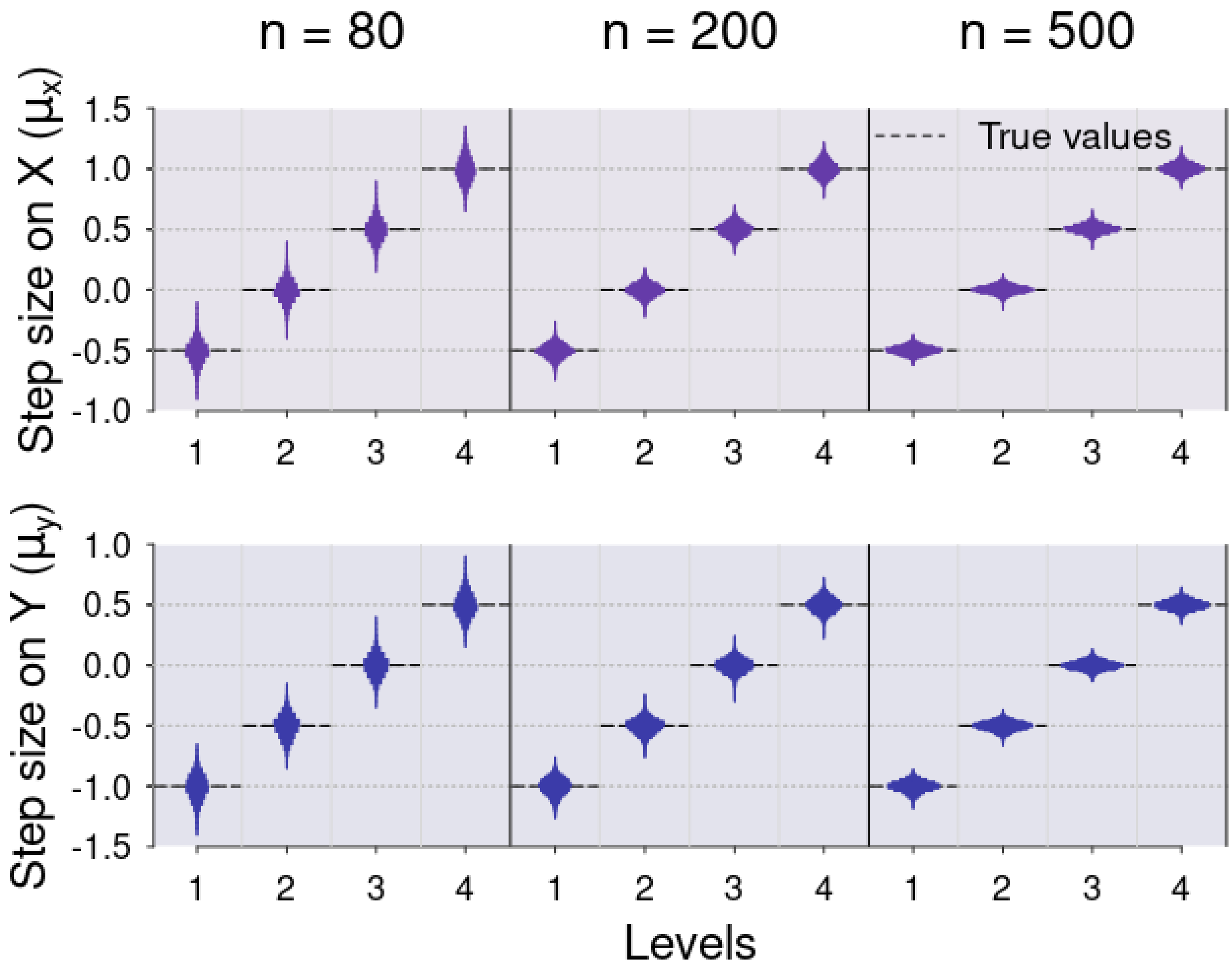
$$\delta \in \{0.01, 1.0, 2.0\}$$

$$\theta \in \{0.0, 2.0, 4.0\}$$

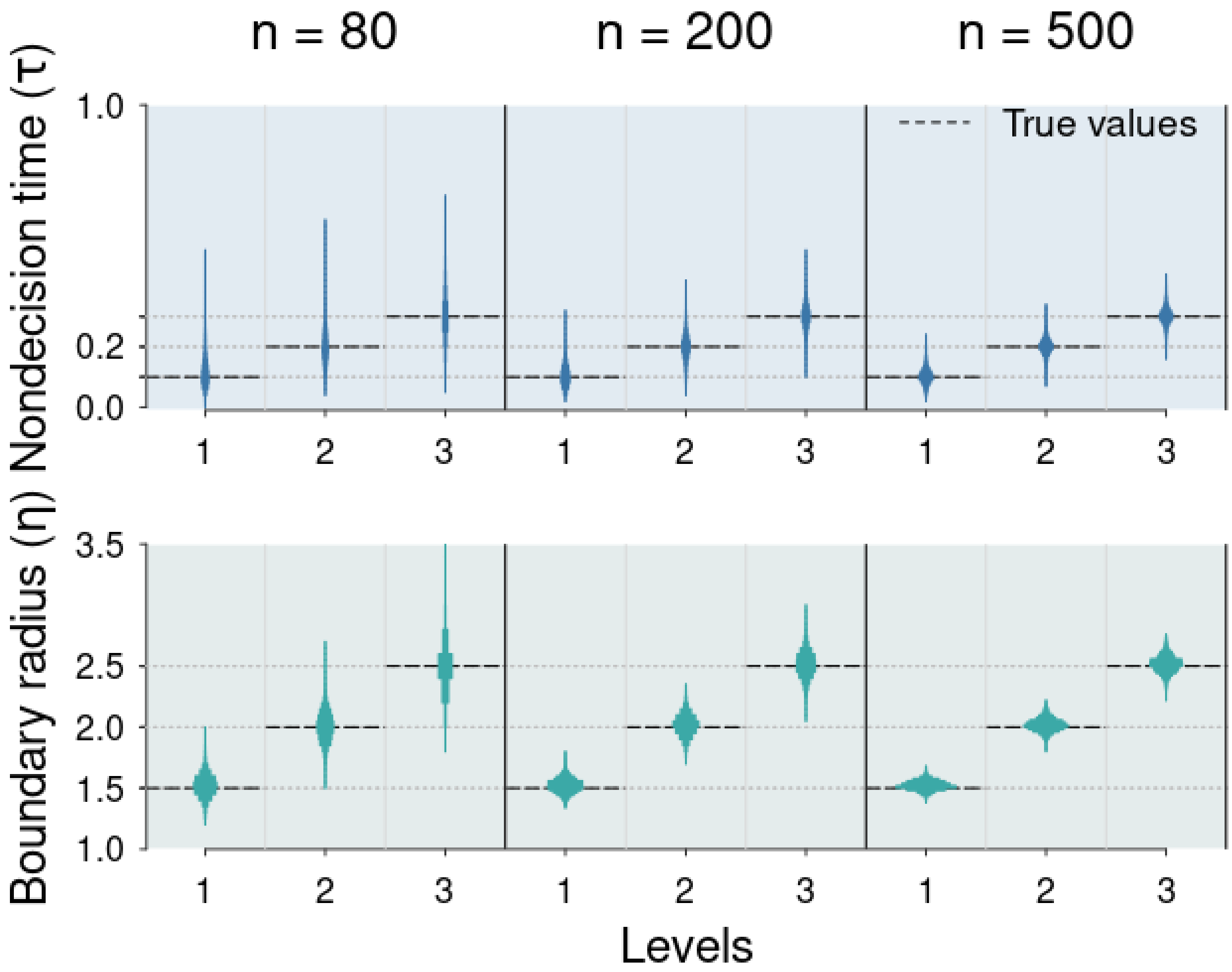
Parameter recovery



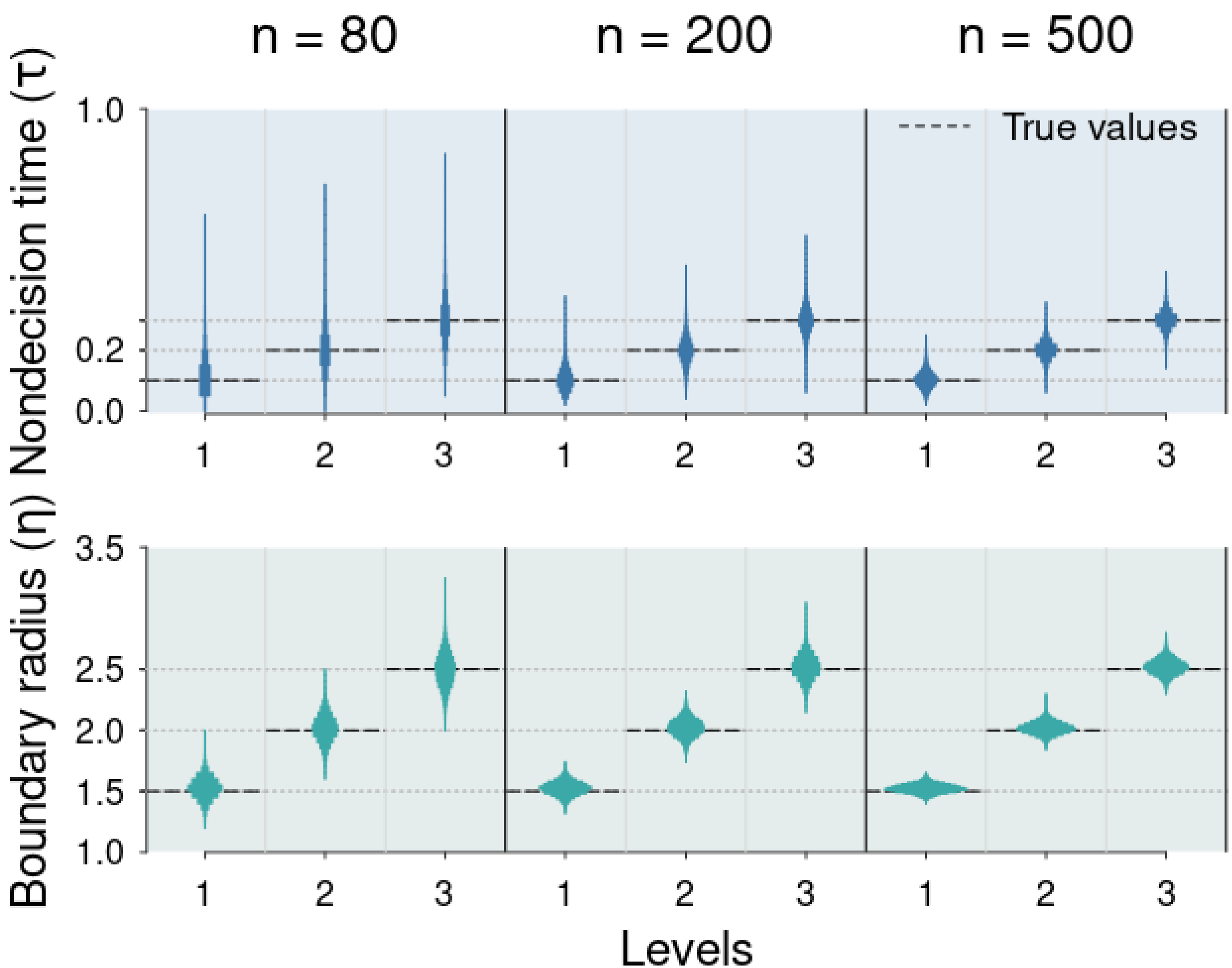
Parameter recovery



Parameter recovery



Parameter recovery



Application on real data

Open data

- Data presented by Peter Kvam (2019).

- Perceptual study where participants produce continuous orientation judgments.

- Task manipulations:

    1. Difficulty levels.
    2. Accuracy vs Speed instructions.
    3. Cue reliability variations.

Task description

- **Stimuli:** A sequence of Gabor patches sampled from a normal distribution.
- **Main instruction:** Indicate the true mean orientation of the patches presented by clicking any point on the circumference of a circle.
- **2 x 2 factorial design:** Speed vs Accuracy - Cued vs Uncued presentation.
- **Three levels of difficulty:** 15, 30 and 45 degrees of standard deviation.

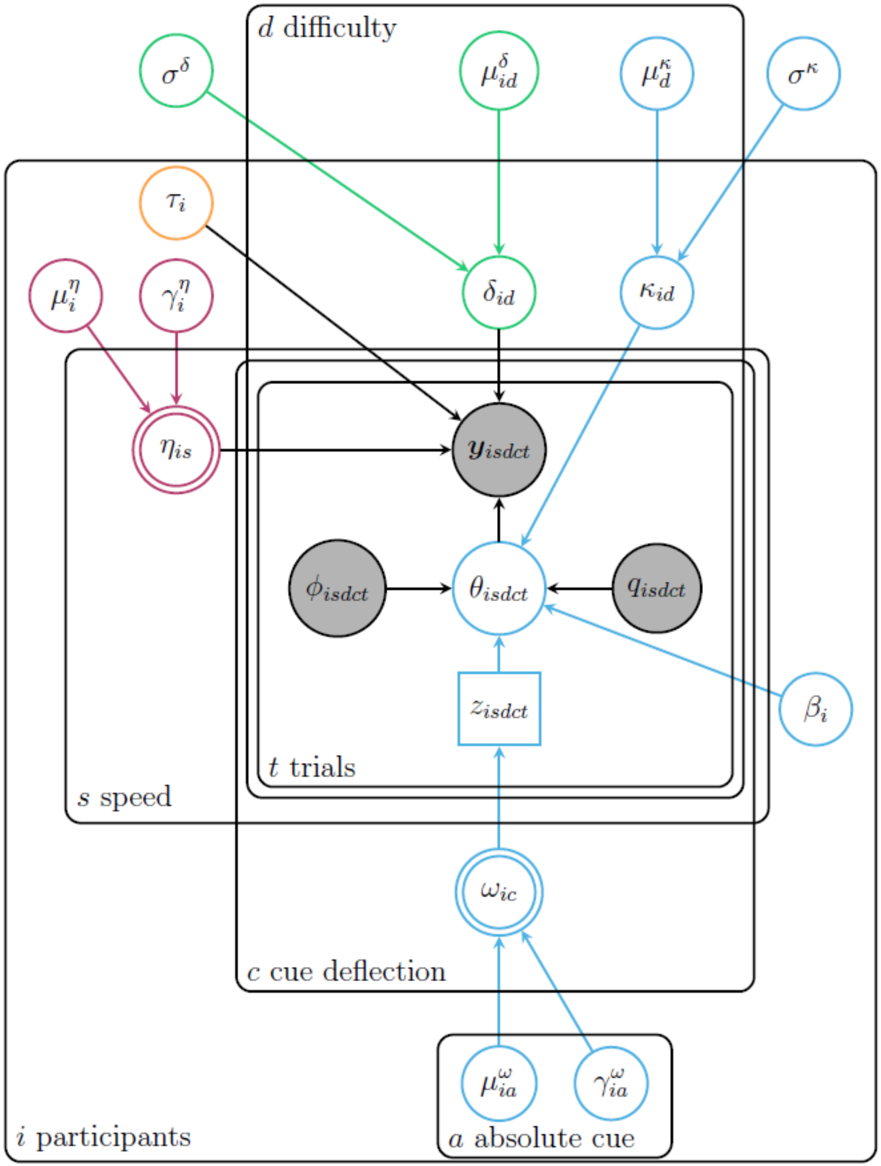
Research questions

1.- Are participants more cautious to respond when they are instructed to prioritize accuracy over speed? (η)

2.- Does information processing speed change with task difficulty? (δ)

3.- Does the consistency of the stimulus information decrease as a function of task difficulty?

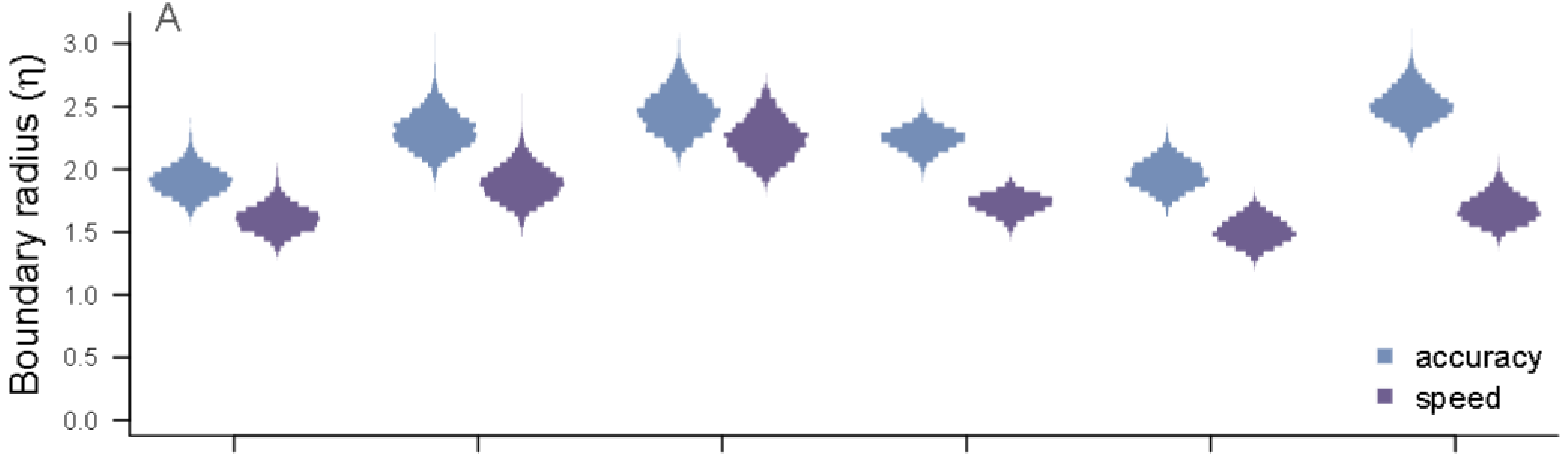
4.- Do different degreees of cue deflections have an impact on how likely participants are to ignore the cue while making a decision?



$$
\begin{aligned}
\mu^\eta_i &\sim \text{Gaussian}(0,1)\\
\gamma^\eta_i &\sim \text{Gaussian}(0,1)T(0,\infty)\\
\eta_{is} &= \begin{cases} \exp\left(\mu^\eta_i + \gamma^\eta_i/2\right) & \text{if } s = \text{accuracy}\\ \exp\left(\mu^\eta_i - \gamma^\eta_i/2\right) & \text{if } s = \text{speed}\end{cases}\\
\tau_i &\sim \text{uniform}(0, \min y_{i1})\\
\mu^\delta_d &\sim \text{Gaussian}(0,1)\\
\sigma^\delta &\sim \text{uniform}(0,1)\\
\delta_{id} &\sim \text{log-Gaussian}\left(\mu^\delta_d, \frac{1}{(\sigma^\delta)^2}\right)\\
\mu^\kappa_d &\sim \text{Gaussian}(0,1)\\
\sigma^\kappa &\sim \text{uniform}(0,4)\\
\kappa_{id} &\sim \text{log-Gaussian}\left(\mu^\kappa_d, \frac{1}{(\sigma^\kappa)^2}\right)\\
\mu^\omega_{ia} &\sim \text{Gaussian}(0,1)\\
\gamma^\omega_{ia} &\sim \text{Gaussian}(0,1)\\
\frac{\omega_{ic}}{1-\omega_{ic}} &= \begin{cases} \exp\left(\mu^\omega_{ia} + \frac{\gamma^\omega_{ia}}{2}\right) & \text{if } c > 0\\ \exp\left(\mu^\omega_{ia} - \frac{\gamma^\omega_{ia}}{2}\right) & \text{if } c < 0\\ \exp\left(\mu^\omega_{ia}\right) & \text{if } c = 0\end{cases}\\
z_{isdct} &\sim \text{Bernoulli}\left(\omega_{ic}\right)\\
\beta_i &\sim \text{uniform}\left(0,1\right)\\
\theta_{isdct} &\sim \begin{cases} \text{Gaussian}\left(\phi_{isdct}, \kappa_{id}\right) & \text{if } z_{isdct} = 0\\ \text{Gaussian}\left(q_{isdct}, \beta_i\kappa_{id}\right) & \text{if } z_{isdct} = 1\end{cases}\\
y_{isdct} &\sim \text{CDDM}_\circ\left(\delta_{id}, \eta_{is}, \tau_i, \text{mod}(\theta_{isdct}, 2\pi)\right)
\end{aligned}
$$

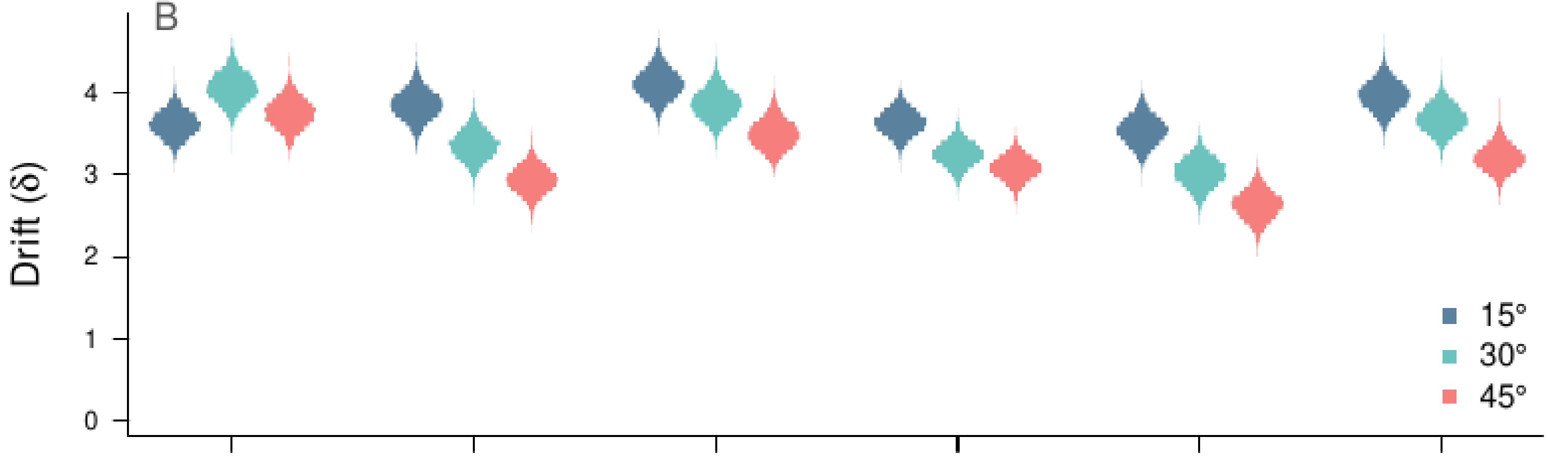
Test 1.

Are participants more cautious to respond in the Accuracy condition?



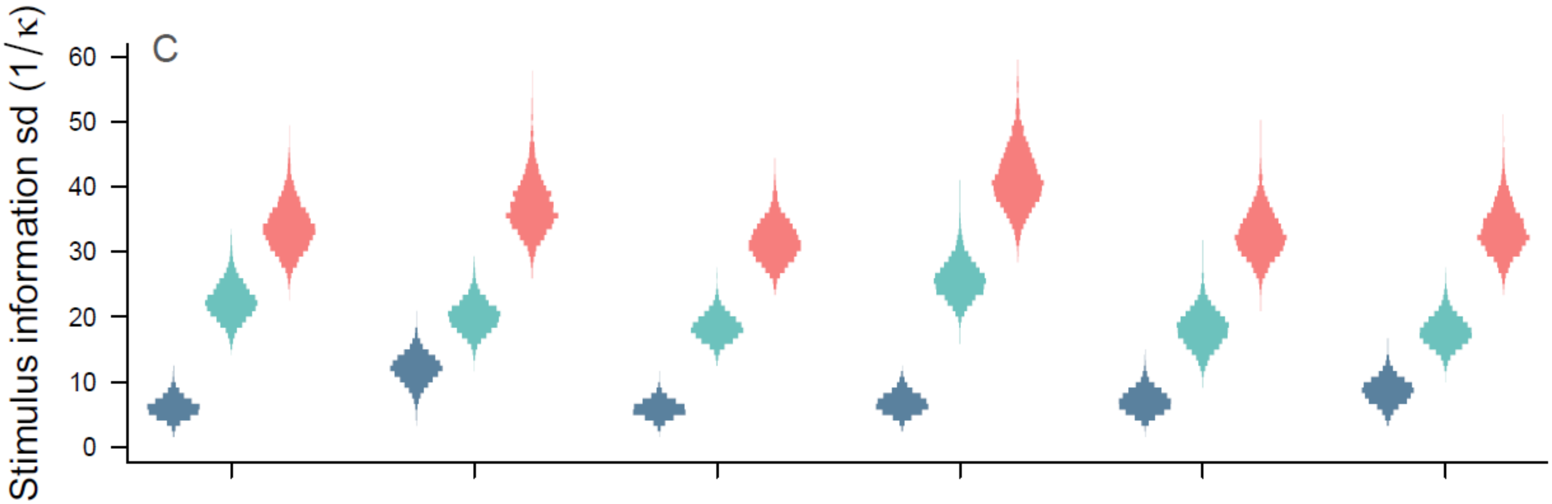
Test 2.

Does information processing speed decrease as task difficulty increases?



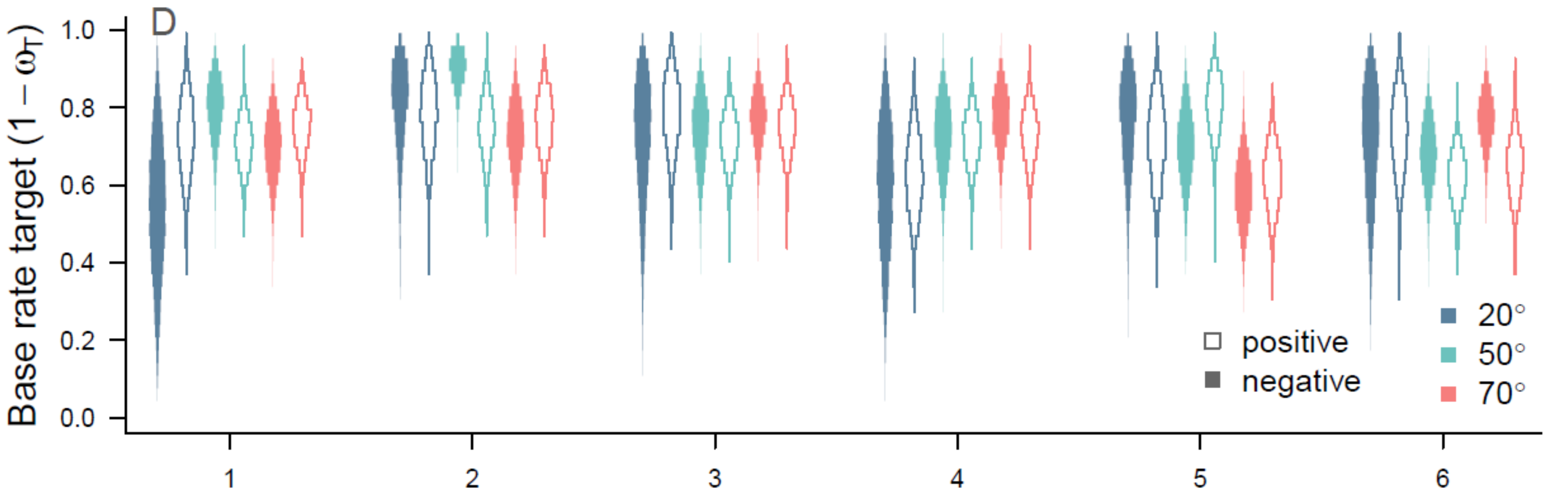
Test 3.

Does the consistency of the stimulus information change as a function of task difficulty?



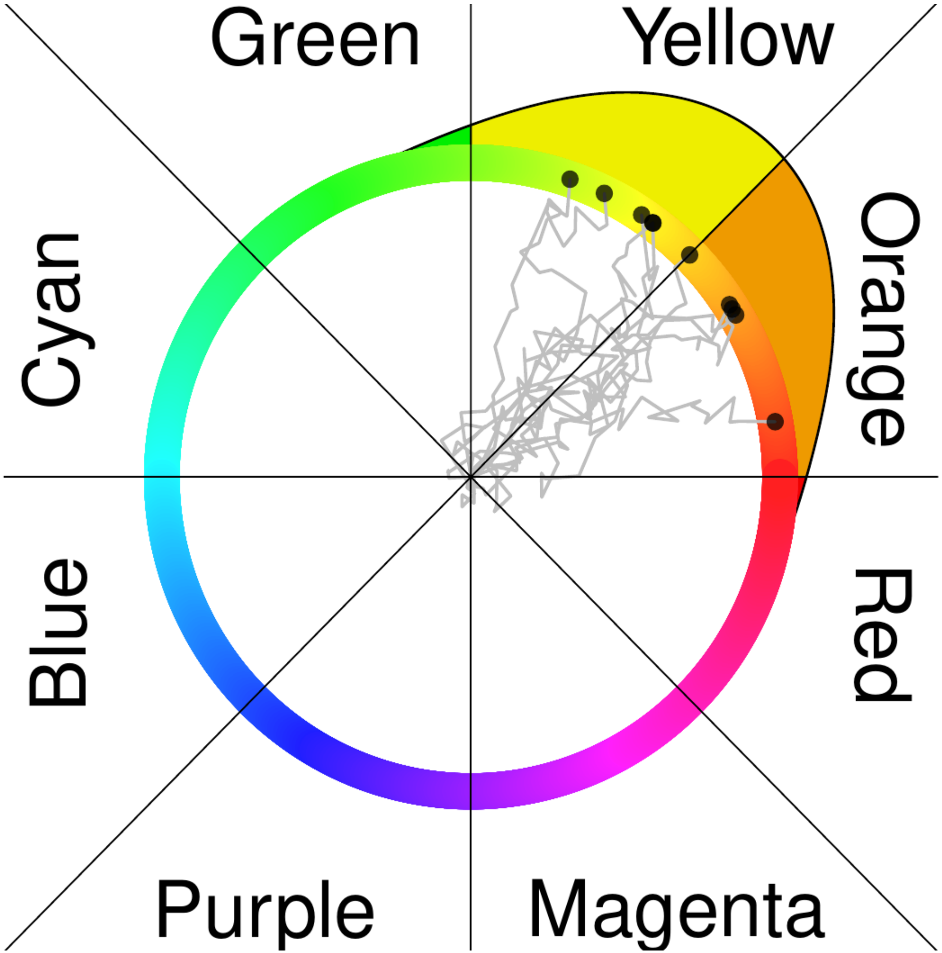
Test 4.

Do different cue deflections change how likely participants are to ignore the cue orientation while making a decision?

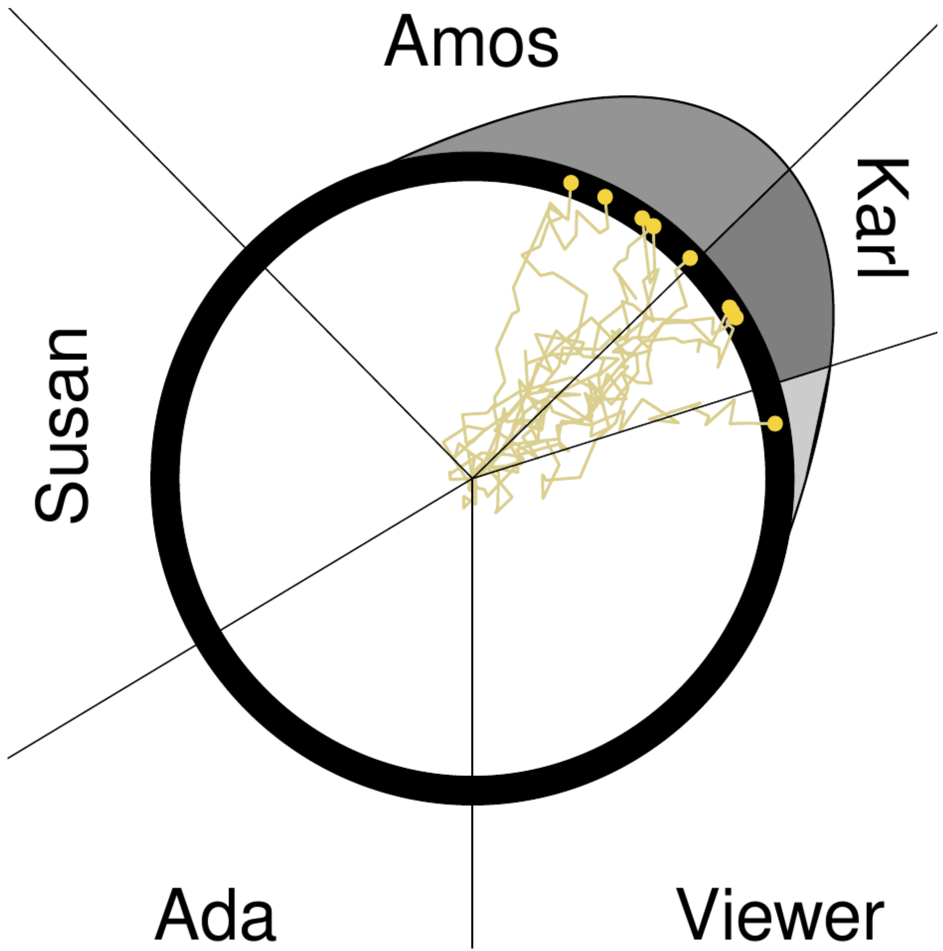


Future work

Discrete extension of the CDDM



Discrete extension of the CDDM



Acknowledgements

Thank you!