An EZ Bayesian hierarchical drift diffusion model for response time and accuracy

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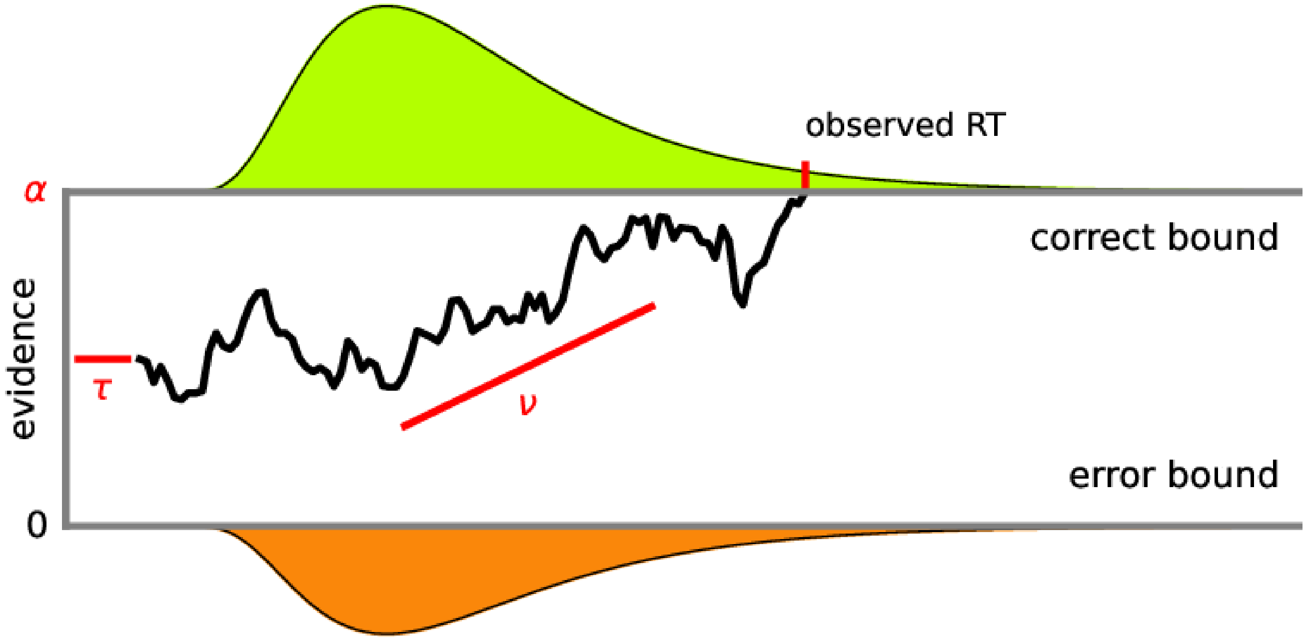
Drift diffusion model (DDM)



$$y \sim \text{Wiener}(\alpha, \nu, \tau)$$

EZ-DDM

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THEORETICAL AND REVIEW ARTICLES

An EZ-diffusion model for response time and accuracy

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The EZ-diffusion model for two-choice response time tasks takes mean response time, the variance of response time, and response accuracy as inputs. The model transforms these data via three simple equations to produce unique values for the quality of information, response conservativeness, and nondecision time. This transformation of observed data in terms of unobserved variables addresses the speed–accuracy trade-off and allows an unambiguous quantification of performance differences in two-choice response time tasks. The EZ-diffusion model can be applied to data-sparse situations to facilitate individual subject analysis. We studied the performance of the EZ-diffusion model in terms of parameter recovery and robustness against misspecification by using Monte Carlo simulations. The EZ model was also applied to a real-world data set.

Forward equations

Let $g = \exp(-\alpha\nu)$.

$$\tilde{R} = \frac{1}{g+1}$$

$$\tilde{M}_{\mathrm{C}} = \tau + \left(\frac{\alpha}{2\nu}\right)\left(\frac{g-1}{g+1}\right)$$

$$\tilde{V}_{\mathrm{C}} = \left(\frac{\alpha}{2\nu^3}\right)\left\{\frac{1-2\alpha\nu g - g^2}{(g+1)^2}\right\}$$

Inverse equations:

Let $L = \log\left(\frac{\dot{R}}{1-\dot{R}}\right)$

$$\hat{\nu} = \operatorname{sign}\left(\dot{R} - \frac{1}{2}\right) \times \cdots$$

$$\cdots \sqrt[4]{\frac{L\left(\dot{R}^2 L - \dot{R}L + \dot{R} - \frac{1}{2}\right)}{\dot{V}_{\mathrm{C}}}}$$

$$\hat{\alpha} = \frac{L}{\hat{\nu}}$$

$$\hat{\tau} = \dot{M}_{\mathrm{C}} - \left(\frac{\hat{\alpha}}{2\hat{\nu}}\right)\left[\frac{1-\exp(-\hat{\nu}\hat{\alpha})}{1+\exp(-\hat{\nu}\hat{\alpha})}\right]$$

Bayesian implementation of the EZ-DDM

- The EZ-DDM provides deterministic estimators $\hat{\nu}$, $\hat{\alpha}$, $\hat{\tau}$.

- We require *probabilistic estimators* and *a distribution over data* that is conditional on the model parameters.

Solution:

- Use the *sampling distributions of the summary statistics* in the EZDDM to build a proxy model

- The proxy model allows for hierarchical Bayesian extensions.

Sampling distributions for the EZ summary statistics

Accuracy rate:

$$\dot{T} \sim \text{Binomial}\left(\tilde{R}, N\right)$$

RT Mean ($\dot{M}$):

$$\dot{M} \sim \text{Normal}\left(\tilde{M}, \frac{\tilde{V}}{N}\right)$$

RT Variance ($\dot{V}$)

$$(N-1)\frac{\dot{V}}{\tilde{V}} \sim \text{Chi-squared}\left(N-1\right)$$

$$\Rightarrow (N-1)\frac{\dot{V}}{\tilde{V}} \sim \text{Gamma}\left(\frac{N-1}{2}, 2\right)$$

$$\Rightarrow \dot{V} \sim \text{Gamma}\left(\frac{N-1}{2}, \frac{2\tilde{V}}{N-1}\right)$$

As $\dot{T}$ becomes sufficiently large:

$$\dot{V} \sim \text{Normal}\left(\tilde{V}, \frac{2\tilde{V}^2}{N-1}\right)$$

The proxy model

$$\dot{T} \sim \text{Binomial}\left(\tilde{R}, N\right)$$

$$\tilde{R} = \frac{1}{g+1}$$

$$\dot{M} \sim \text{Normal}\left(\tilde{M}, \frac{\tilde{V}}{N}\right)$$

$$\tilde{M} = \tau + \left(\frac{\alpha}{2\nu}\right)\left(\frac{g-1}{g+1}\right)$$

$$\dot{V} \sim \text{Normal}\left(\tilde{V}, \frac{2\tilde{V}^2}{N-1}\right)$$

$$\tilde{V} = \left(\frac{\alpha}{2\nu^3}\right)\left\{\frac{1-2\alpha\nu g - g^2}{(g+1)^2}\right\}$$

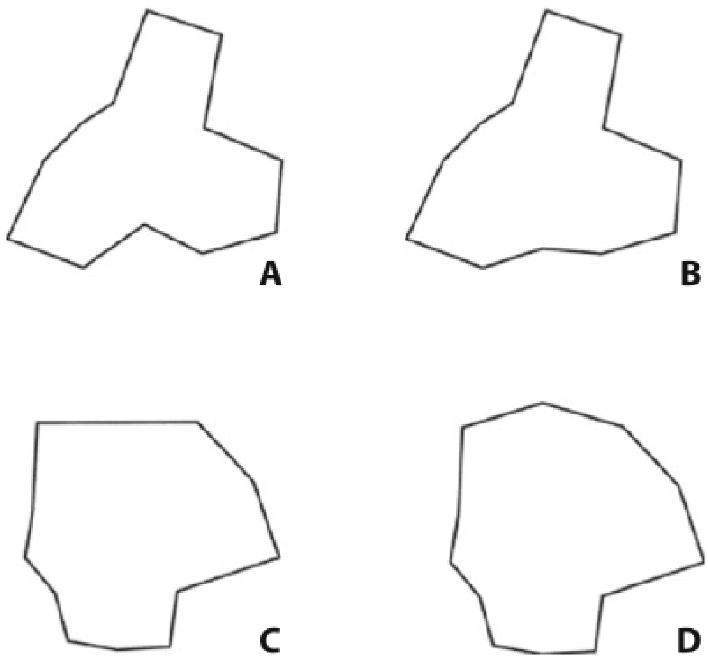
Hypothesis testing example

Data: Shape perception study by Vandekerckhove, Panis, and Wagemans (2007)

Task: Are the images shown on screen same or different?

Design:

- **Change occurs?** Yes / No
- **Change type:** Change in concavity vs convexity
- **Change quality:** Qualitative vs Quantitative change



The model

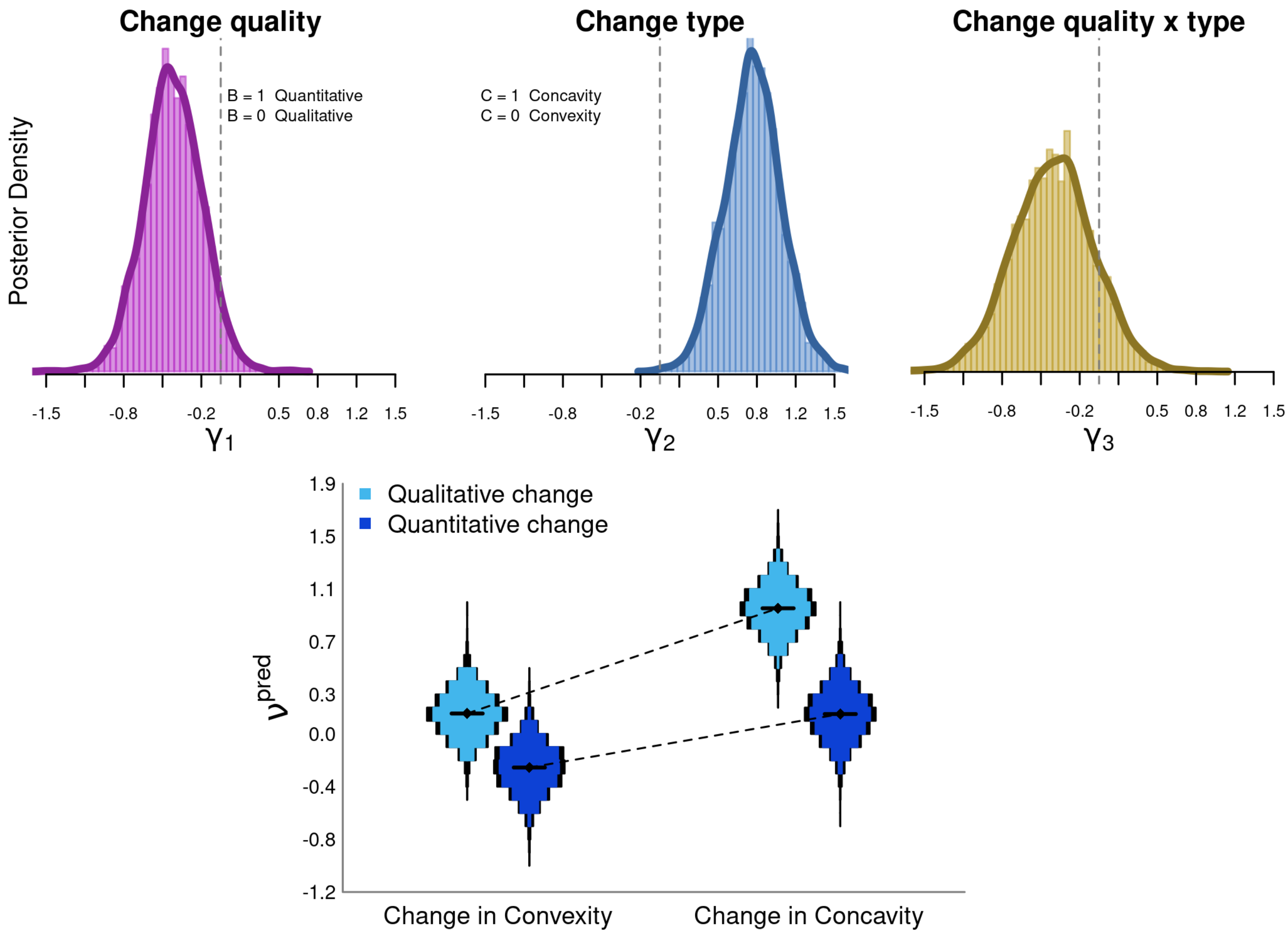
We fit a multiple linear regression on ν

$$\nu \sim \mathrm{Normal}(\nu_k^{pred}, \sigma_\nu)$$

So that for every condition k, the predicted drift rate is determined by the configuration of three dummy variables A, B and C.

$$\nu_k^{pred} = \mu + A_k(\gamma_1 B_k + \gamma_2 C_k + \gamma_3 B_k C_k) + (1 - A_k)\gamma_4$$

Condition	Change (A)	Change quality (B)	Change type (C)
k = 1	A = 1 Yes	B = 0 Qualitative	C = 0 Convexity
k = 2	A = 1 Yes	B = 1 Quantitative	C = 0 Convexity
k = 3	A = 1 Yes	B = 0 Qualitative	C = 1 Concavity
k = 4	A = 1 Yes	B = 1 Quantitative	C = 1 Concavity
k = 5	A = 0 No		



Estimation time: **4.05s** ($N = 5,760$, `iter = 1000`, and `burnin = 100`).

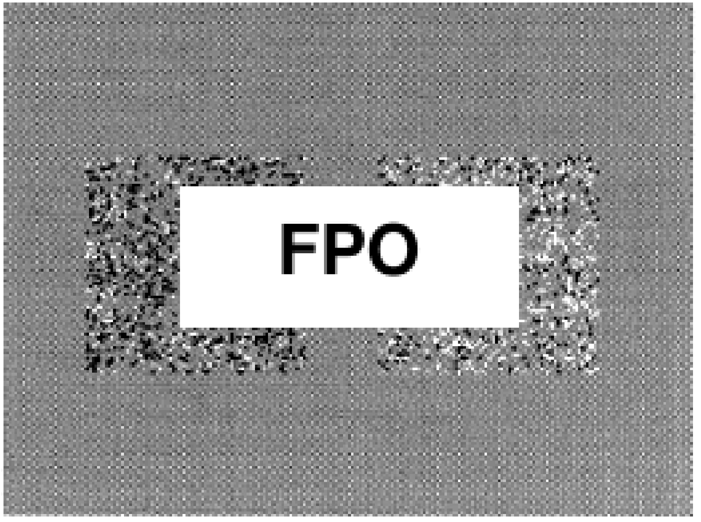
Metaregression example

Data: Numerosity study by Ratcliff and Rouder, 1998.

Task: Is the overall brightness of pixel arrays displayed on the monitor “high” or “low”?

Design:

- **Instruction conditions:** Speed vs Accuracy.
- **Pixel array levels:** 16 "more white" and 16 "more black" levels.



The model

- An effect (β) of instruction (i.e., x_i) on α.

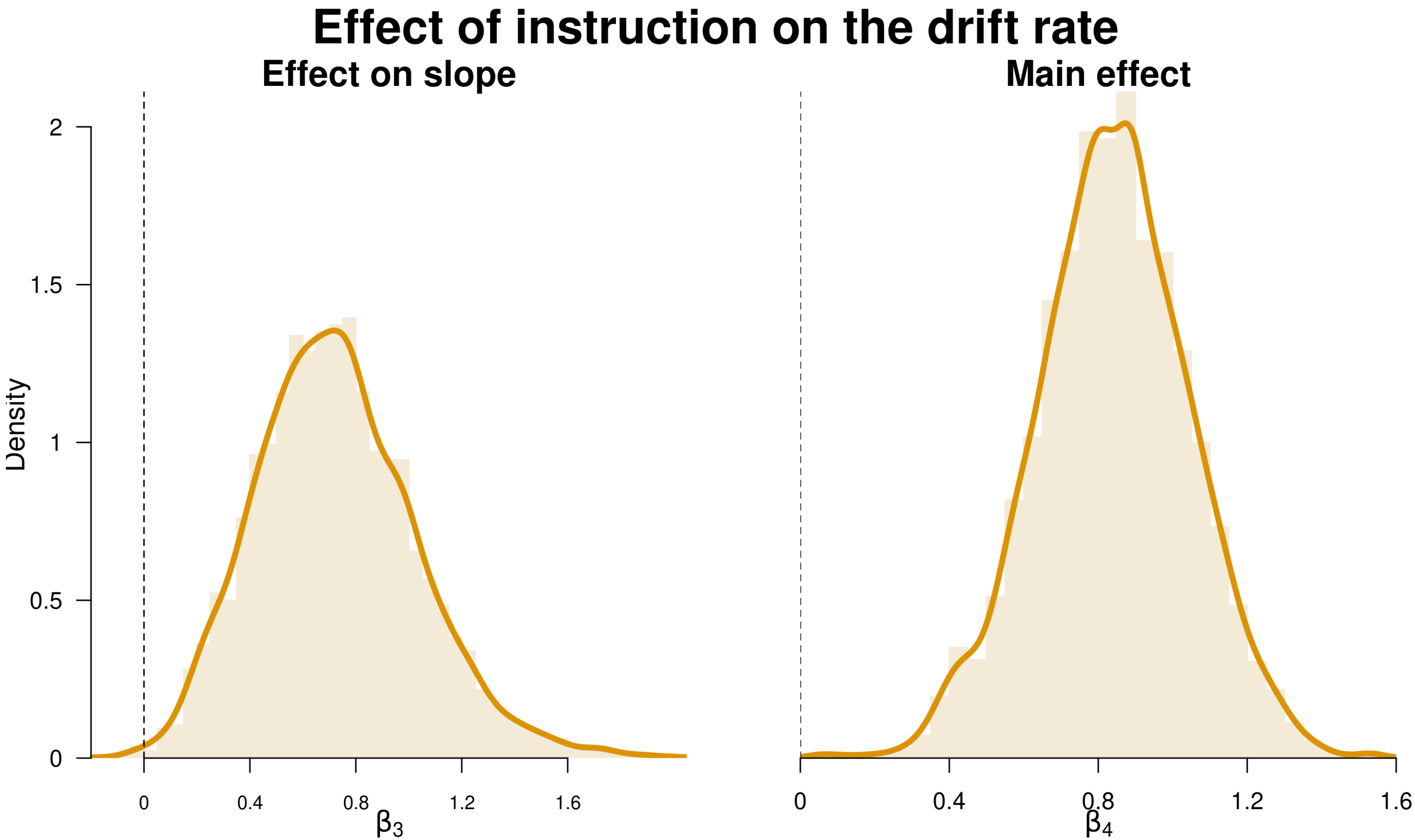
$$\alpha \sim \text{Normal}(\mu_\alpha + \beta X_i, \sigma_\alpha)$$

- A nonlinear regression on ν using instruction (i.e., x_i) and stimulus configuration (i.e., x_s) as predictors.

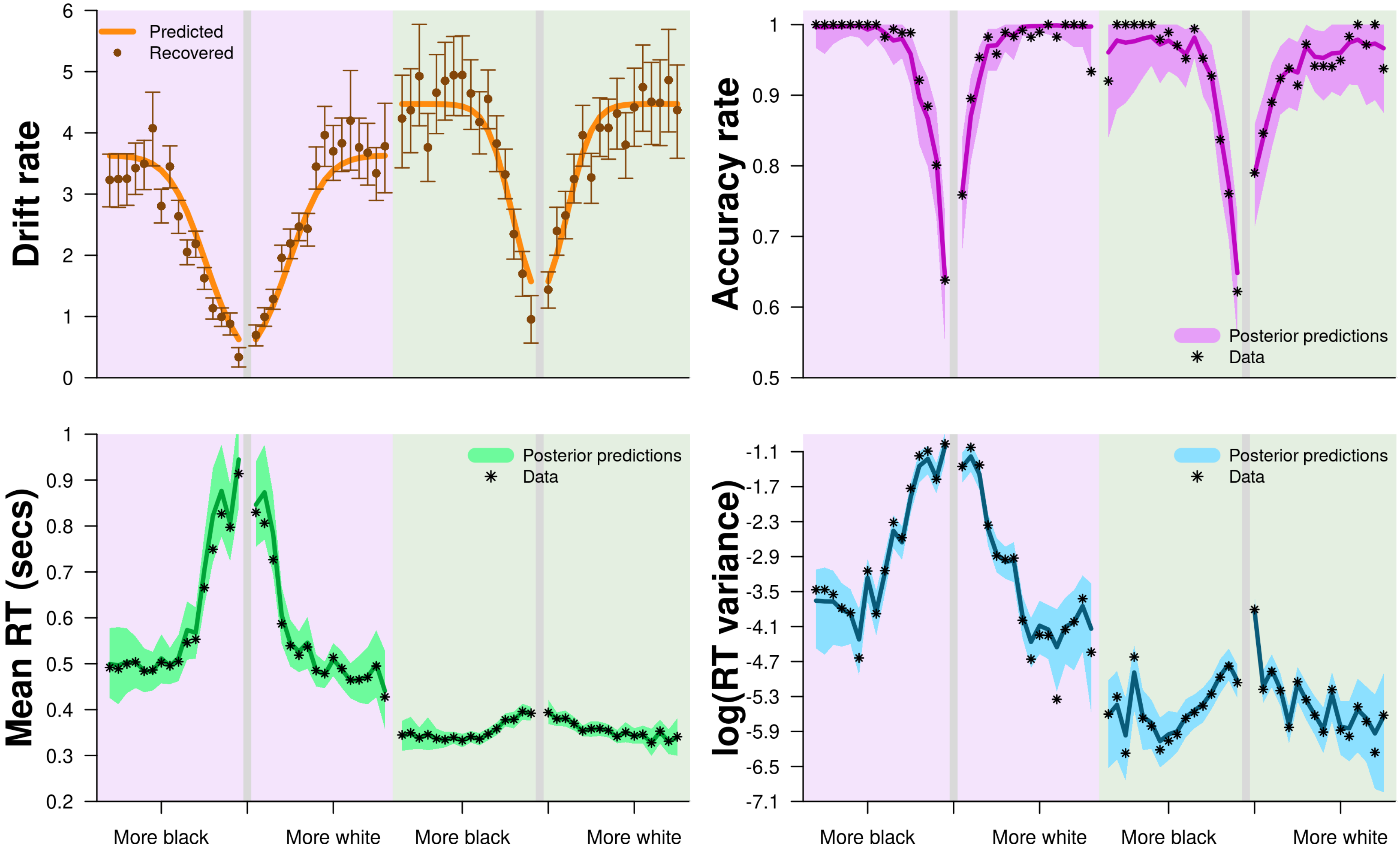
$$Q_{i,s} = \Phi(\beta_1 + \beta_2 |X_s| + \beta_3 X_i |X_s|)$$

$$\nu_{i,s}^{pred} = \mu_\nu + \beta_0 Q_{i,s} + \beta_4 X_i$$

$$\nu_{i,s} \sim \text{Normal}(\nu_{i,s}^{pred}, \sigma_\nu)$$



Estimation time: **8.58s** ($N = 7,889$, `iter = 1000`, and `burnin = 100`).



Drift rate
Predicted
Recovered
Accuracy rate
Posterior predictions
Data
Mean RT (secs)
Posterior predictions
Data
log(RT variance)
Posterior predictions
Data
More black
More white
More black
More white
More black
More white
More black
More white

Closing remarks

- This proxy model facilitates the Bayesian implementation of the EZDDM, which can be extended hierarchical to account for the structure of the data.

- The EZBHDDM can be implemented in any probabilistic programming language.

- The EZBHDDM is hyper-efficient: it takes only seconds to run!

- The EZBHDDM is able to recover hierarchical and regression parameters.

Thank you!

(I'm looking for a post-doc position!)

Pre-print: