Bayesian Cognitive and Statistical Modeling Applied to Signal Detection Theory and the Mirror Effect in a Perceptual Task

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Introduction

The **mirror effect** is a well-established empirical result in *recognition memory*: when subjects' performance is compared between two classes of stimuli, one known to be easier to recognize (A class) than the other (B class), this difference is reflected in the identification of both target and lure stimuli (Glanzer et al., 1993). Its name comes from the suggested order of the underlying distributions according to SDT.

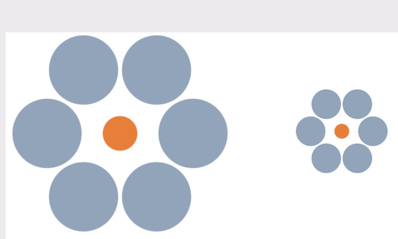
$$FA(A) < FA(B) < Hits(B) < Hits(A) \qquad (1)$$

Method: A Perceptual Task

To assess the generalizability of the mirror effect, we designed a **perceptual SD-task** (*no study phase included*), where what's known about the Ebbinghaus illusion led to construction of the two classes of stimuli, A and B (Massaro & Anderson 1971):

- A class (*"easy"*): 2 or 3 surrounding circles - Weak illusion
- B class (*"hard"*): 7 or 8 surrounding circles - Strong illusion

1. **Detection Task:** Are the central circles the same size?
   - Experiment 1: Just one Ebbinghaus illusion on screen.
   - Experiment 2: Both circles were Ebbinghaus illusions.



2. **Confidence Rating:** How certain are you of your previous response? (1-3 scale)

Replicating the Original Data Analysis

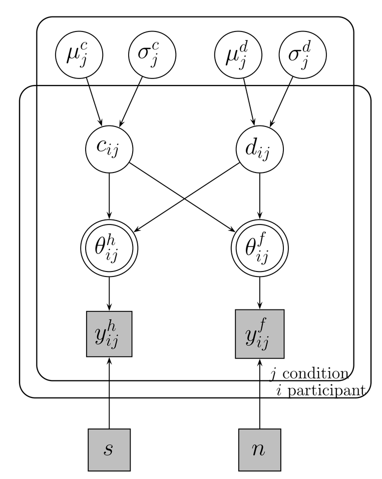
We conducted a step by step replication of the mean-based analysis reported in the literature (Glanzer & Adams, 1990), and found that:

1. Differences among d' are statistically significant ($d'(A) > d'(B)$)
2. Differences among response rates are significant too ($H(A) > H(B)$ & $FA(A) < FA(B)$)
3. Differences among ratings are significant ($H(A) > H(B)$ & $FA(A) < FA(B)$)

A Bayesian Approach

Given the probabilistic nature of the SDT model, it seems like the study of the mirror effect can benefit from the application of Bayesian statistical and cognitive modeling to evaluate the differences observed in the performance of participants across each class of stimuli.

- We apply a Hierarchical SDT model

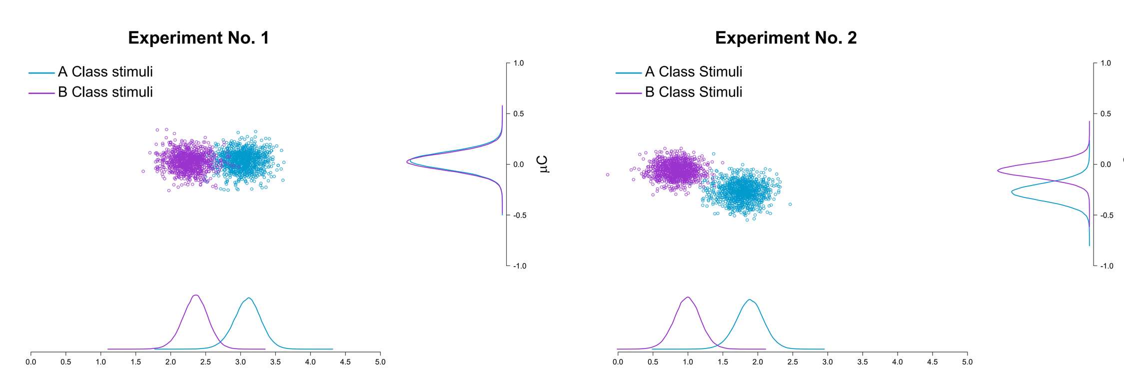


$$\mu_j^d \sim \text{Gaussian}(0,1)_{T(0,6)}$$
$$\mu_j^c \sim \text{Gaussian}(0,0.7)$$
$$\sigma_j^c, \sigma_j^d \sim \text{Uniform}(0,4)$$

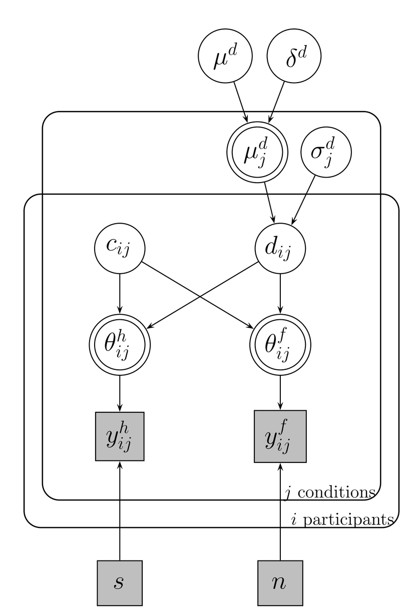
$$d_{ij} \sim \text{Gaussian}(\mu_j^d, \sigma_j^d)$$
$$c_{ij} \sim \text{Gaussian}(\mu_j^d, \sigma_j^c)$$

$$\theta_{ij}^h = \phi(\tfrac{1}{2}d_{ij} - c_{ij})$$
$$\theta_{ij}^f = \phi(-\tfrac{1}{2}d_{ij} - c_{ij})$$

$$y_{ij}^h \sim \text{Binomial}(\theta_{ij}^h, s)$$
$$y_{ij}^f \sim \text{Binomial}(\theta_{ij}^f, n)$$



- We test the differences in d' across classes of stimuli



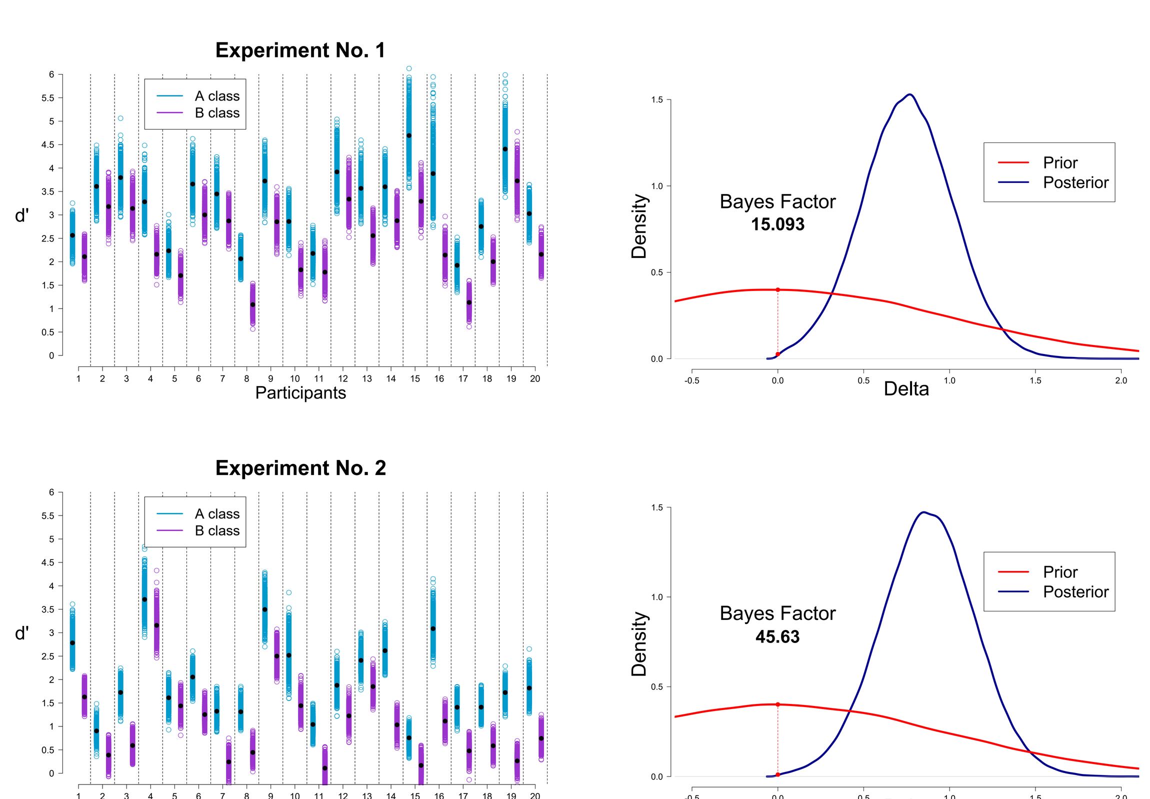
$$\mu^d \sim \text{Gaussian}(0,1)_{T(0,6)}$$
$$\delta^d \sim \text{Gaussian}(0,1)_{T(0,6)}$$

$$\mu_A^d = \mu^d + \tfrac{\delta}{2}$$
$$\mu_B^d = \mu^d - \tfrac{\delta}{2}$$
$$\sigma_j^d \sim \text{Uniform}(0,5)$$

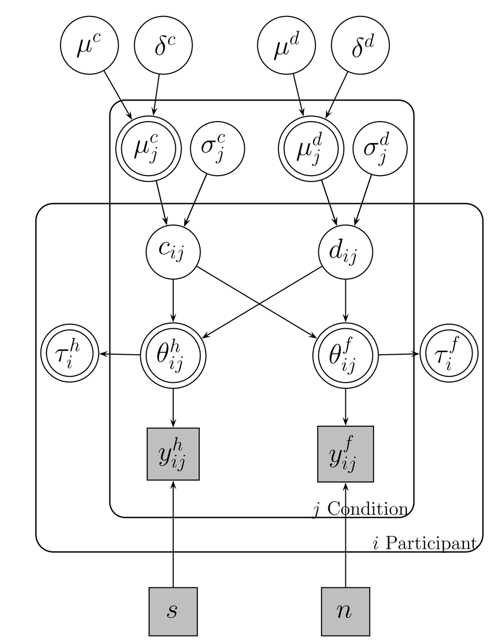
$$d_{ij} \sim \text{Gaussian}(\mu_j^d, \sigma_j^d)$$
$$c_{ij} \sim \text{Gaussian}(0,0.7)$$

$$\theta_{ij}^h = \phi(\tfrac{1}{2}d_{ij} - c_{ij})$$
$$\theta_{ij}^f = \phi(-\tfrac{1}{2}d_{ij} - c_{ij})$$

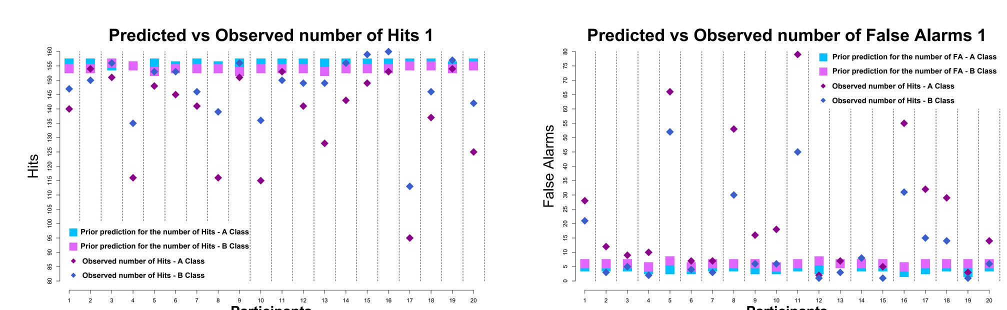
$$y_{ij}^h \sim \text{Binomial}(\theta_{ij}^h, s)$$
$$y_{ij}^f \sim \text{Binomial}(\theta_{ij}^f, n)$$

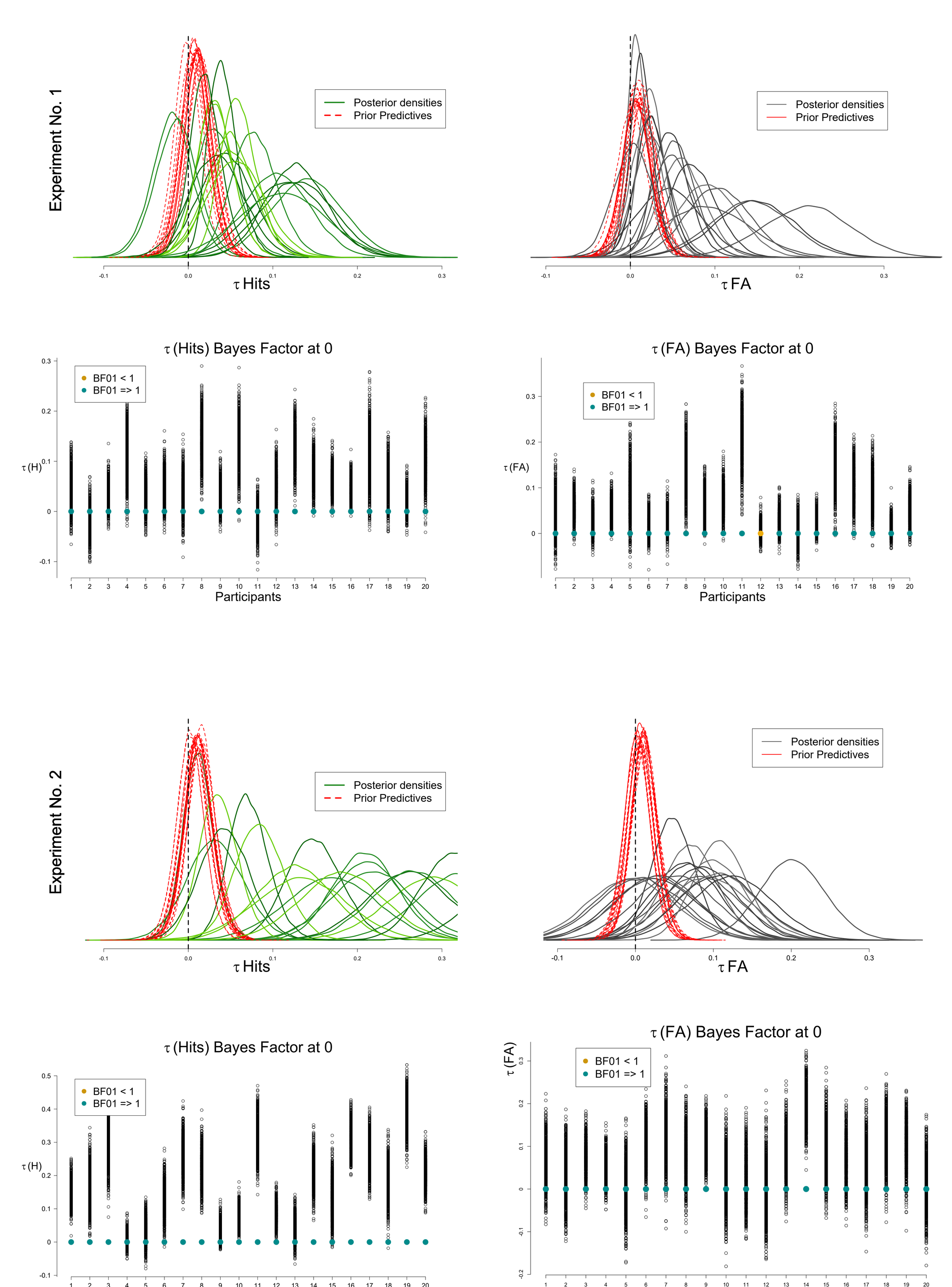


- Are the Hit and FA rates different across A and B?

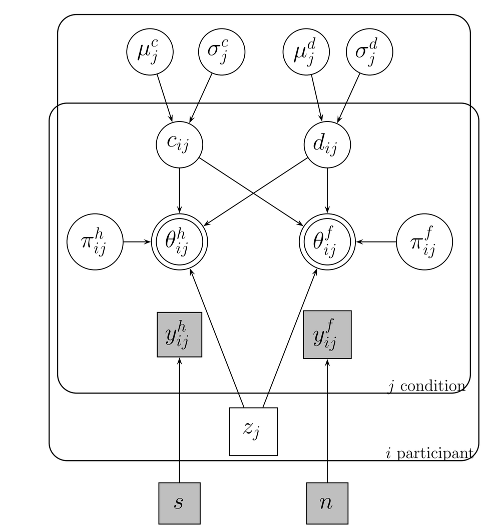


$$\mu^c \sim \text{Gaussian}(0,0.7)$$
$$\delta^c \sim \text{Gaussian}(0,0.3)$$
$$\mu_A^c = \mu^c + \frac{\delta^c}{2}$$
$$\mu_B^c = \mu^c - \frac{\delta^c}{2}$$
$$\mu^d \sim \text{Gaussian}(0,1)_{T(0,6)}$$
$$\delta^d \sim \text{Gaussian}(0,1)_{T(0,6)}$$
$$\mu_A^d = \mu^c + \frac{\delta^d}{2}$$
$$\mu_B^d = \mu^c - \frac{\delta^d}{2}$$
$$\sigma_j^c, \sigma_j^d \sim \text{Uniform}(0,5)$$
$$d_{ij} \sim \text{Gaussian}(\mu_j^d, \sigma_j^d)$$
$$c_{ij} \sim \text{Gaussian}(\mu_j^c, \sigma_j^c)$$
$$y_{ij}^h \sim \text{Binomial}(\theta_{ij}^h, s)$$
$$y_{ij}^f \sim \text{Binomial}(\theta_{ij}^f, s)$$
$$\theta_{ij}^h = \phi(\tfrac{1}{2}d_{ij} - c_{ij})$$
$$\theta_{ij}^f = \phi(-\tfrac{1}{2}d_{ij} - c_{ij})$$
$$\tau_i^h = \theta_{iA}^h - \theta_{iB}^h \qquad \tau_i^f = \theta_{iB}^f - \theta_{iA}^f$$





Contaminant Model



$$\mu_j^c \sim \text{Gaussian}(0,0.7)$$
$$\mu_j^d \sim \text{Gaussian}(0,1)_{T(0,6)}$$
$$\sigma_j^c, \sigma_j^d \sim \text{Uniform}(0,4)$$

$$d_{ij} \sim \text{Gaussian}(\mu_j^d, \sigma_j^d)$$
$$c_{ij} \sim \text{Gaussian}(\mu_j^c, \sigma_j^c)$$

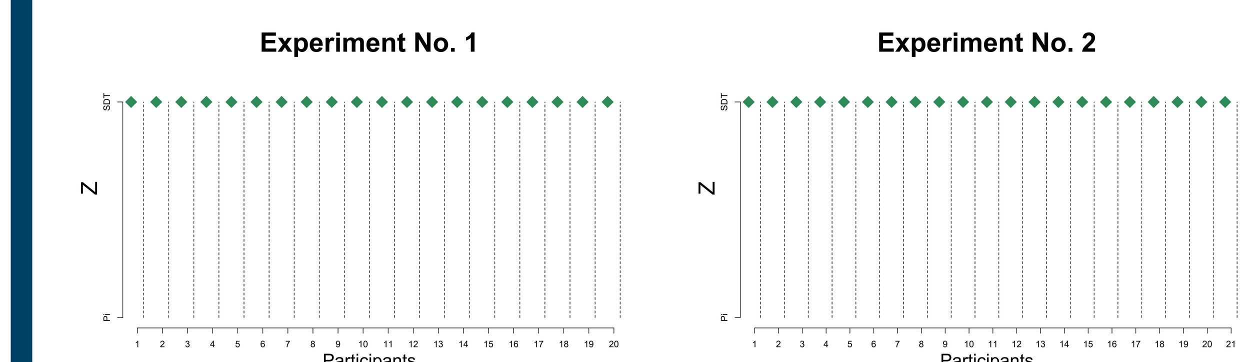
$$\pi_{ij}^h, \pi_i^f \sim \text{Beta}(1,1)$$

$$\theta_{ij}^h \leftarrow \begin{cases} \pi_{ij}^h & \text{if } z_j = 0 \\ \phi(\tfrac{1}{2}d_{ij} - c_{ij}) & \text{if } z_j = 1 \end{cases}$$

$$\theta_{ij}^f \leftarrow \begin{cases} \pi_{ij}^f & \text{if } z_j = 0 \\ \phi(-\tfrac{1}{2}d_{ij} - c_{ij}) & \text{if } z_j = 1 \end{cases}$$

$$z_j \sim \text{Bernoulli}(0.5)$$

$$y_{ij}^h \sim \text{Binomial}(\theta_{ij}^h, s)$$
$$h_{ij}^f \sim \text{Binomial}(\theta_{ij}^f, s)$$



Discussion

The present study showed evidence of the mirror effect in a SD-task that didn't involve recognition memory. The perceptual task here presented lacked a pre-experimental phase where participants had the chance to change the maginitude of the illusions associated with each class A andB. This suggests that there might be a much more basic principle regulating the mirror effect pattern of responses.

The application of Bayesian cognitive and statistical modeling allowed us to assess this phenomenon at the individual level, preserving the probabilistic nature assumed by SDT.

References

- Glanzer, M., Adams, J. (1990) The Mirror Effect in Recognition Memory Data and Theory.*Journal of Experimental Psychology: Learning, Memory and Cognition, 16* (1), 5-16.
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- Massaro, D., Anderson, N. (1971). Judgmental model of the Ebbinghaus Illusion. *Journal of Experimental Psychology, 89*, 147 - 151.

Acknowledgments & Contact Information

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