An EZ Bayesian Hierarchical Drift-Diffusion Model for Response Time and Accuracy

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Overview

1. The **Simple DDM** is a three-parameter model that accounts for choice and RT data using a Wiener likelihood function.
2. The **EZ-DDM** provides parameter estimators from summary statistics.
3. We present a probabilistic formulation of the EZ-DDM based on the sampling distributions of these summary statistics, allowing for a **hierarchical EZ-DDM** that can be implemented in a **Bayesian** framework.

The EZ-DDM

The Simple DDM describes choices and RTs in binary choice tasks as the result of a hidden evidence accumulation process defined by three parameters:

Parameter	Interpretation
Drift rate ν	Evidence accumulation rate
Boundary α	Distance between response bounds
Nondecision time τ	Encoding and motor control time

$$\mathbf{y} \sim \text{Wiener}\left(\nu, \alpha, \tau\right)$$

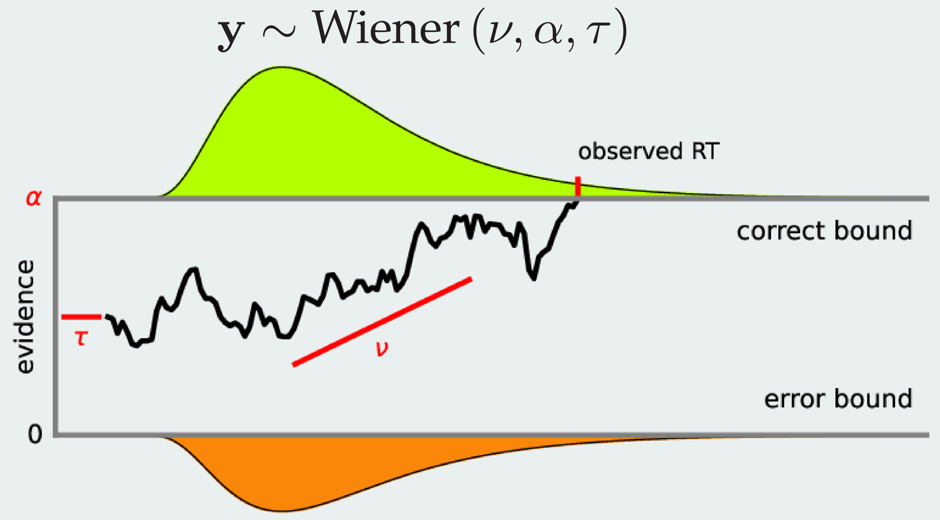

Figure 1. Illustration of the Simple DDM.

Using the method of moments, we can express the accuracy rate ($A^{\text{pred}}_{\text{mean}}$), and the mean and variance of the RTs ($RT^{\text{pred}}_{\text{mean}}$ and $RT^{\text{pred}}_{\text{var}}$) as closed-form functions of the model parameters [3].

$$A^{\text{pred}}_{\text{mean}} = \frac{1}{1+\exp(-\alpha\nu)} \tag{1}$$

$$RT^{\text{pred}}_{\text{mean}} = \tau + \left(\frac{\alpha}{2\nu^3}\right)\left[\frac{\exp(-\alpha\nu)-1}{\exp(-\alpha\nu)+1}\right] \tag{2}$$

$$RT^{\text{pred}}_{\text{var}} = \left(\frac{\alpha}{2\nu^3}\right)\left\{\frac{-2\alpha\nu\exp(-\alpha\nu)-\exp(-2\alpha\nu)+1}{[\exp(-\alpha\nu)+1]^2}\right\} \tag{3}$$

The EZ-DDM formulation results from inverting equations 1, 2, and 3 to obtain parameter estimators from these same summary statistics.

Bayesian Hierarchical EZDDM

We present a **proxy model for hierarchical Bayesian EZ-DDMs** built from what we know about the sampling distributions of $RT^{\text{obs}}_{\text{mean}}$ and $RT^{\text{obs}}_{\text{var}}$, and by casting $A^{\text{pred}}_{\text{mean}}$ as the probability of a correct response used to model the total number of correct responses observed ($A^{\text{obs}}_{\text{total}}$) using a binomial distribution:

$$A^{\text{obs}}_{\text{total}} \sim \text{Binomial}\left(A^{\text{pred}}_{\text{mean}}, N\right) \tag{4}$$

$$RT^{\text{obs}}_{\text{mean}} \sim \text{Normal}\left(RT^{\text{pred}}_{\text{mean}}, \frac{RT^{\text{pred}}_{\text{var}}}{N}\right) \tag{5}$$

$$RT^{\text{obs}}_{\text{var}} \sim \text{Normal}\left(RT^{\text{pred}}_{\text{var}}, \frac{2\left[RT^{\text{pred}}_{\text{var}}\right]^2}{N-1}\right). \tag{6}$$

Together with equations 1, 2, and 3, equations 4, 5, and 6 present generative distributions for choice and RT summary statistics. This allows us to build **hyper-efficient Bayesian hierarchical extensions of the EZ-DDM** that can be implemented on any probabilistic programming language (e.g., JAGS, STAN, etc.), and which take **only seconds to run!**

Cognitive Psychometrics

Cognitive models use parameters to formalize psychological assumptions about the processes generating the data observed. **Cognitive psychometrics** proposes the **use of cognitive models as psychological measurement tools**. This endeavor benefits from building Bayesian hierarchical extensions that explain variability across individual parameters as a function of covariates and predictors of interest. For example, we can model the data $\mathbf{y}_{i,p}$ from participant p in trial i as:

$$\mathbf{y}_{i,p} \sim \text{Wiener}(\nu_p, \alpha_p, \tau_p)$$

with individual α_p and τ_p sampled from population-level distributions:

$$\alpha_p \sim \text{Normal}\left(\mu_\alpha, \sigma^2_\alpha\right) \qquad \tau_p \sim \text{Normal}\left(\mu_\tau, \sigma^2_\tau\right)$$

and a meta-regression on ν_p that allows us to explore the effect β of a covariate of interest x_p.

$$\nu_p \sim \text{Normal}\left(\mu_\nu + \beta x_p, \sigma^2_\nu\right)$$

Applied Example 1

Task description: We analyze data from one participant in a study [1] where participants had to judge the brightness of pixel arrays as "high" or "low."

- 33 stimulus configuration levels controlling the proportion of black vs. white pixels.
- 2 instruction conditions: Speed vs. Accuracy.

Background: The instruction condition is known to have an effect on the boundary parameter α.

Question: Does the instruction condition have an effect on the drift rate ν?

Model: We use a nonlinear regression model to explore the effects of instruction X_i and stimulus configuration Z_s on $\nu_{i,s}$:

$$S_{i,s} = \Phi(\beta_1 + \beta_2|Z_s| + \beta_3 X_i|Z_s|)$$
$$\nu^{\text{pred}}_{i,s} = \mu_\nu + \beta_0 S_{i,s} + \beta_4 X_i$$
$$\nu_{i,s} \sim \text{Normal}(\nu^{\text{pred}}_{i,s}, \sigma_\delta)$$

Results and conclusions:

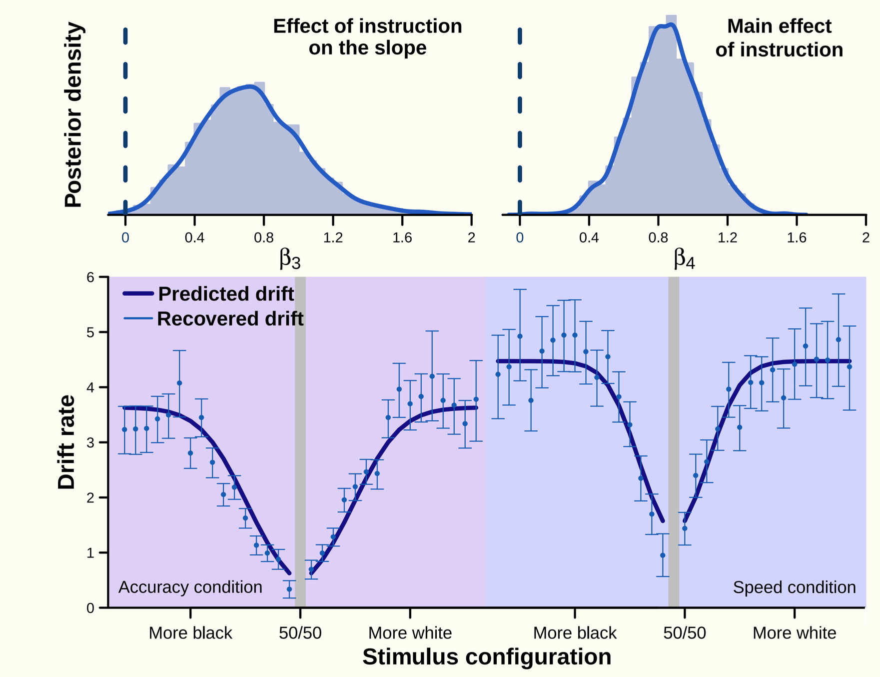

Figure 2. Results from Applied Example 1. Detailed description below.

1. The instruction condition has a **main effect** and an **effect on the drift rate slope** (Fig. 2, top panel).
2. The predicted and recovered drift rate per difficulty level **changes across instruction conditions** (Fig. 2, bottom panel).

This model took approximately 8 seconds to run on 7,802 trials.

Applied Example 2

Task description: We analyze data from a shape perception study [2] where participants had to indicate whether two irregular shapes interspersed with a mask were the "same" or "different." The experimental design used three factors to define five experimental conditions $k \in \{1 \ldots 5\}$.

	Change (A)	Change quality (B)	Change type (C)
$k=1$	Yes ($A=1$)	Qualitative ($B=0$)	Convexity ($C=0$)
$k=2$	Yes ($A=1$)	Quantitative ($B=1$)	Convexity ($C=0$)
$k=3$	Yes ($A=1$)	Qualitative ($B=0$)	Concavity ($C=1$)
$k=4$	Yes ($A=1$)	Quantitative ($B=1$)	Concavity ($C=1$)
$k=5$	No ($A=0$)	n/a	n/a

Background: Changes in concavity are known to be easier to detect that changes in convexity.

Question: Does the quality of the change mediate the effect of change type?

Model: We use a multiple linear regression model to explore the differences across ν_k.

$$\nu^{\text{pred}}_k = \mu + A_k(\gamma_1 B_k + \gamma_2 C_k + \gamma_3 B_k C_k) + (1-A_k)\gamma_4$$
$$\nu_k \sim \text{Normal}(\nu^{\text{pred}}_k, \sigma_\nu).$$

Results and conclusions:

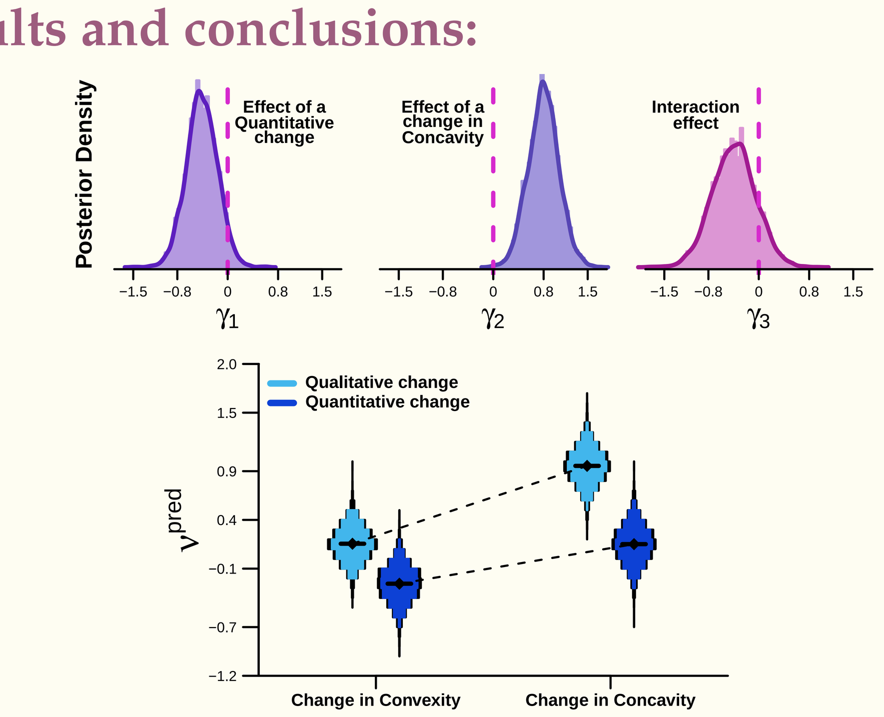

Figure 3. Results from Applied Example 2. Detailed description below.

1. We confirm that **changes in concavity are easier to detect** than changes in convexity (Fig. 3, top left).
2. We find that **qualitative changes are easier to detect** than quantitative changes.
3. **There is an interaction between change type and quality.** The difference in difficulty between a qualitative and quantitative change is larger in changes in concavity.

This model took approximately 4 seconds to run on 5,760 trials.

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Preprint cidlab.com/paper/102

References

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Acknowledgments

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